1. Let $R$ be the region bounded by

$$y = x^2 + 2 \quad \text{and} \quad y = 3x^2$$

Set up, but do not evaluate, an integral which represents the volume of each of the following solids:

A picture always helps:

The intersections points can be found by setting $x^2 + 2 = 3x^2 \implies 2 = 2x^2 \implies x = \pm 1$.

(a) **(2 points)** The solid obtained by rotating $R$ about the $x$-axis.

We are rotating the region about the $x$-axis, so we get

As the picture indicates, it is easier to use washers than shells. The “big radius” $R$ is $x^2 + 2$. The “small radius” $r$ is $3x^2$. The $x$-values we want run from $x = -1$ to $x = 1$, so the volume is represented by

$$\int_{-1}^{1} \pi(x^2 + 2)^2 - \pi(3x^2)^2 \, dx$$

(b) **(2 points)** The solid obtained by rotating $R$ about the line $x = 2$.

We are rotating the region about the line $x = 2$, so we get
We could use washers to answer this question, but as indicated by the picture, it is easier to use shells. Draw the height at $x$, and rotate about the line $x = 2$. The radius is $2 - x$, and the height is $x^2 + 2 - 3x^2 = 2 - x^2$. The $x$-values that we want run from $x = -1$ to $x = 1$, so the volume is represented by

$$
\int_{-1}^{1} 2\pi(2 - x)(2 - x^2) \, dx
$$

(c) **(2 points)** The solid obtained by rotating $R$ about the line $y = 3$. We are rotating the region about the line $y = 3$, so we get

As the picture indicates, it is easier to use washers than shells. The “big radius” $R$ is $3 - 3x^2$. The “small radius” $r$ is $3 - (x^2 + 2) = 1 - x^2$. The $x$-values we want run from $x = -1$ to $x = 1$, so the volume is represented by

$$
\int_{-1}^{1} \pi(3 - 3x^2)^2 - \pi(1 - x^2)^2 \, dx
$$
(d) **(2 points)** The solid whose base is $R$ and whose cross-sections, perpendicular to the $x$-axis, are semi-circles.

The base of this solid is

![Graph of a function](image)

The blue line in the above picture is the diameter of a cross-section semi-circle at $x$. Thus, the diameter of the cross-section semi-circle is $x^2 + 2 - 3x^2 = 2 - 2x^2$, so the radius of the cross-section semi-circle is $(2 - 2x^2)/2 = 1 - x^2$ and the area of the cross-section semi-circle is

$$\pi r^2/2 = \frac{\pi}{2} (1 - x^2)^2$$

so the volume is

$$\int_{-1}^{1} \frac{\pi}{2} (1 - x^2)^2 \, dx$$

2. **(2 points)** Precisely state the Mean Value Theorem.

Let $f$ be a function which is continuous on $[a,b]$ and differentiable on $(a,b)$. Then there is a number $c$ in the interval $(a,b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$