1. (3 points) Let $R$ be the region bounded by the $x$-axis and the graph of $y = \ln x$ on the interval $[2, 3]$. Set up, but do not evaluate, the definite integral which represents the volume of the solid obtained when $R$ is revolved around the line $x = 1$. Use proper notation.

$V = \int_0^2 \pi(2^2 - y^2) \, dy + \int_2^3 \pi(3^2 - (e^y)^2) \, dy$

2. (3 points) Set up, but do not evaluate, the definite integral which represents the volume of the solid obtained by rotating the region bounded by the curves $y = 4 - x^2$ and the $x$-axis about the $x$-axis.

$V = \int_{-2}^2 A(x) \, dx = \int_{-2}^2 \pi(4 - x^2)^2 \, dx$
3. (2 points) Precisely state Rolle's Theorem.

If \( f \) is a function satisfying
(1) \( f \) is continuous on the closed interval \([a,b]\)
(2) \( f \) is differentiable on the open interval \((a,b)\)
(3) \( f(a) = f(b) \)

where \( a \neq b \), then there is a number \( c \) with \( a < c < b \)
\[ f'(c) = 0. \]

4. (2 points) Explain carefully why \( f(x) = x^5 + 5x^3 + 3x - 10 \) cannot have two real roots.

Say \( f \) does have two real roots, \( a, b \), so \( f(a) = f(b) = 0 \).
Without any loss of generality, we can assume \( a < b \).
Now \( f \) is a polynomial, hence it is cont \& diff everywhere. So in particular, it is cont \& diff on \([a,b]\) and \((a,b)\) respectively. Then by Rolle's Theorem, there must be a \( c \) with \( a < c < b \) s.t. \( f'(c) = 0 \).

Now let's evaluate \( f''(x) \): \( f'(x) = 5x^4 + 15x^2 + 3 \).
\( 5x^4 \geq 0 \) and \( 15x^2 \geq 0 \) for every \( x \). Hence \( f''(x) = 20x^2 + 3 \geq 3 \geq 0 \). In particular for any \( c \) with \( a < c < b \), \( f''(c) > 0 \).
This contradicts our conclusion from Rolle's Theorem.
Therefore our initial assumption must be false. So \( f \) does not have two real roots.