Math 220 (section BD2)  
Quizz 10 
Spring 2012

Name ________________________________

• No calculators allowed.
• Show sufficient work to justify each answer.
• You have 15 minutes for this quiz.

1. (2pts) Let \( R \) be the region bounded by the curves \( y = e^x \), \( x = 1 \), \( x = 2 \), \( y = 0 \). Set up but do not evaluate the definite integral which represents the volume of the solid obtained by rotating \( R \) around the \( y \)-axis.

\[
V = \int_{1}^{2} 2\pi x e^x \, dx
\]

2. (3pts) Let \( R \) be the region bounded by the curves \( y = \sqrt{x} \), \( y = \frac{x}{2} \). Set up but do not evaluate the definite integral which represents the volume of the solid obtained by rotating \( R \) around the \( x \)-axis.

\[
\frac{x}{2} = \sqrt{x} \\
\frac{x^2}{4} = x \\
\frac{x^2}{4} - x = 0 \\
x\left(\frac{x}{4} - 1\right) = 0 \\
\frac{x}{4} - 1 = 0 \\
\frac{x}{4} = 1 \Rightarrow x = 4
\]

\[
V = \int_{0}^{4} \left[ \pi \left( \sqrt{x} \right)^2 - \pi \left( \frac{x}{2} \right)^2 \right] \, dx
\]
3. (2pts) Precisely state the Mean Value Theorem.

Let \( f \) be a function that satisfies

(a) \( f \) is continuous on the closed interval \([a, b]\)

(b) \( f \) is differentiable on the open interval \((a, b)\)

Then there is a number \( c \) in \((a, b)\) such that

\[
f'(c) = \frac{f(b) - f(a)}{b - a}
\]

4. (2pts) Let \( f(x) = e^x \). Find the number \( c \) that satisfies the conclusion of the Mean Value Theorem on the interval \([0, 1]\).

Notice \( f \) is cont. and diff. on \([0, 1]\) so MVT holds.

\[
\begin{align*}
    f'(x) &= e^x \\
    f(1) &= e \\
    f(0) &= 1
\end{align*}
\]

\[
\Rightarrow \frac{f(1) - f(0)}{1 - 0} = e - 1
\]

\[
e^c = e - 1 \quad \Leftrightarrow \quad \ln(e^c) = \ln(e - 1) \quad \Rightarrow \quad c = \ln(e - 1)
\]

5. (1pt) Assume that the equation \( x^3 + 4x = 0 \) has at least one solution. Explain carefully why the equation cannot have more than one solution.

**1st Soln.** Let \( f(x) = x^3 + 4x \). Suppose there are two distinct numbers \( r_1, r_2 \) such that \( f(r_1) = 0 \), \( f(r_2) = 0 \).

Then by Rolle's theorem on \([r_1, r_2]\) there must exist \( c \in (r_1, r_2) \) such that \( f'(c) = 0 \). However \( f'(x) = 3x^2 + 4 > 0 \) is positive for all \( x \) so \( f'(c) = 0 \) is impossible hence \( f \) cannot have two distinct roots.

**2nd Soln.** \( x^3 + 4x = x(x^2 + 4) = 0 \)

The polynomial \( x^2 + 4 \) is always positive hence the only solution to the equation is \( x = 0 \).