1. Let \( R \) be the region bounded by
\[
y = x^2 + 1 \quad \text{and} \quad y = 2x^2
\]

Set up, but do not evaluate, an integral which represents the volume of each of the following solids:
A picture always helps:

The intersections points can be found by setting \( x^2 + 1 = 2x^2 \Rightarrow 1 = x^2 \Rightarrow x = \pm 1. \)

(a) \textbf{(2 points)} The solid obtained by rotating \( R \) about the line \( x = 2. \)

We are rotating the region about the line \( x = 2, \) so we get

We could use washers to answer this question, but as indicated by the picture, it is easier to use shells. Draw the height at \( x, \) and rotate about the line \( x = 2. \) The radius is \( 2 - x, \) and the height is \( x^2 + 1 - 2x^2 = 1 - x^2. \) The \( x \)-values that we want run from \( x = -1 \) to \( x = 1, \) so the volume is represented by
\[
\int_{-1}^{1} 2\pi(2 - x)(1 - x^2) \, dx
\]
(b) **(2 points)** The solid obtained by rotating $R$ about the $x$-axis.

We are rotating the region about the $x$-axis, so we get

As the picture indicates, it is easier to use washers than shells. The “big radius” $R$ is $x^2 + 1$. The “small radius” $r$ is $2x^2$. The $x$-values we want run from $x = -1$ to $x = 1$, so the volume is represented by

$$\int_{-1}^{1} \pi (x^2 + 1)^2 - \pi (2x^2)^2 \, dx$$

(c) **(2 points)** The solid obtained by rotating $R$ about the line $y = 2$.

We are rotating the region about the line $y = 2$, so we get

As the picture indicates, it is easier to use washers than shells. The “big radius” $R$ is $2 - 2x^2$. The “small radius” $r$ is $2 - (x^2 + 1) = 1 - x^2$. The $x$-values we want run from $x = -1$ to $x = 1$, so the volume is represented by

$$\int_{-1}^{1} \pi (2 - 2x^2)^2 - \pi (1 - x^2)^2 \, dx$$
(d) **(2 points)** The solid whose base is $R$ and whose cross-sections, perpendicular to the $x$-axis, are semi-circles.

The base of this solid is

![Diagram](image)

The blue line in the above picture is the diameter of a cross-section semi-circle at $x$. Thus, the diameter of the cross-section semi-circle is $x^2 + 1 - 2x^2 = 1 - x^2$, so the radius of the cross-section semi-circle is $(1 - x^2)/2$ and the area of the cross-section semi-circle is

$$\pi r^2 / 2 = \pi \left( \frac{1 - x^2}{2} \right)^2$$

so the volume is

$$\int_{-1}^{1} \frac{\pi}{2} \left( \frac{1 - x^2}{2} \right)^2 \, dx$$

2. **(2 points)** Precisely state the Mean Value Theorem.

Let $f$ be a function which is continuous on $[a, b]$ and differentiable on $(a, b)$. Then there is a number $c$ in the interval $(a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$