1. (2 points)
Show that \( f(x) = 4x^3 + 15x - 6 \) has exactly one real root.

\[
\begin{align*}
f(1) &= 4 + 15 - 6 = 13 > 0 \\
f(0) &= -6 < 0
\end{align*}
\]
Then by IVT there is a root. \( \checkmark \)

If there are two roots, by MVT there is a \( c \in \mathbb{R} \) such that \( f'(c) = 0 \)

\[
\begin{align*}
f'(x) &= 12x^2 + 15 \\
f'(c) &= 0 = 12c^2 + 15
\end{align*}
\]
\( \int \) no real number satisfies

Then, there is exactly one root.

2. (2 points)
Let \( R \) be the finite region bounded by \( y = 3 - x \) and the \( x \)-axis and \( y \)-axis. Set up, but do not evaluate, a definite integral which represents the volume of the solid obtained when \( R \) is revolved around the horizontal line \( y = 5 \).

\[
\int_0^3 \pi R^2 - \pi r^2 \, dx
\]
\( R = 5 \)
\( r = 5 - y = 5 - (3 - x) = 2 + x \)

\[
\int_0^3 \pi 5^2 - \pi (x+2)^2 \, dx
\]
3. (3 points each) Let $R$ be the finite region bounded by $y = x^3 + 2$, $y = 4$ and the $y$-axis. In the following manner set up, but do not evaluate, definite integrals which represent the volume of the solid obtained when $R$ is revolved around the $y$-axis.

(a) Integrate with respect to $x$.

\[
\begin{align*}
4 &= x^3 + 2 \\
\int_{0}^{\sqrt[3]{2}} 2\pi x \, h \, dx \\
\text{here } h &= 4 - (x^3 + 2) \\
\int_{0}^{\sqrt[3]{2}} 2\pi x \, (4 - x^3 - 2) \, dx
\end{align*}
\]

(b) Integrate with respect to $y$. (The integrands in parts (a) and (b) should be different.)

\[
\begin{align*}
\int_{2}^{4} \pi r^2 \, dy \\
\text{same graph} \\
2 \leq y \leq 4 \\
y = x^3 + 2 \\
x^3 = y - 2 \\
x = \sqrt[3]{y - 2} \\
\int_{2}^{4} \pi (y - 2)^{2/3} \, dy
\end{align*}
\]