Name: 

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 15 minutes for this quiz.

1. (2 points) Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Find all numbers \( c \) that satisfy the conclusion.

\[ f(x) = x^3 - 3x + 2 \text{ on } [-2, 2] \]

\( \text{(i) } f(x) \text{ is continuous everywhere, so } f(x) \text{ is continuous on } [-2, 2] \)

\( \text{(ii) } f(x) \text{ is differentiable everywhere so } f'(x) \text{ is differentiable on } [-2, 2] \)

We have satisfied both hypotheses, so the MVT tells us there exists a \( c \) in \((-2, 2)\) such that

\[ f'(c) = \frac{f(2) - f(-2)}{2 - (-2)} \]

\[ f'(c) = \frac{4 - 0}{4} = 1 \]

\[ f''(c) = 3x^2 - 3 \]

\[ 1 = 3c^2 - 3 \]
\[ 3c^2 = 4 \]
\[ c^2 = 4/3 \]
\[ c = \pm \frac{2}{\sqrt{3}} \]

Both are in \([-2, 2]\), therefore when \( c = \frac{2}{\sqrt{3}} \)

the conclusion as MVT is satisfied.
2. Consider the region bounded by the curves, \( y = x \) and \( y = \frac{x^2}{2} \). Intersect at \( x = 0 \) and \( x = 2 \).

(a) (2 points) Set up but **do not solve** the integral for the volume of the solid obtained by rotating the region about the \( x \)-axis and integrating with respect to \( x \).

Rotating around \( x \)-axis \( \Rightarrow \) Cross sections perpendicular to \( x \) \( \Rightarrow \) Integrate w.r.t \( x \)

\[
\text{larger radius} = x \quad \text{smaller radius} = \frac{x^2}{2}
\]

\[
\int_0^2 \pi (x)^2 - \pi \left( \frac{x^2}{2} \right)^2 \, dx = \pi \int_0^2 x^2 - \frac{x^4}{4} \, dx
\]

(b) (2 points) Set up but **do not solve** the integral for the volume of the solid obtained by rotating the region about the \( x \)-axis and integrating with respect to \( y \).

Shells \( \Rightarrow \) w.r.t \( y \)

\[
\text{radius} = y \quad \text{height} = \sqrt{2y} - y
\]

\[
\int_0^2 2\pi (y)(\sqrt{2y} - y) \, dy = 2\pi \int_0^2 y(\sqrt{2y} - y) \, dy
\]

(c) (2 points) Set up but **do not solve** the integral for the volume of the solid obtained by rotating the region about the \( y \)-axis (you may integrate with respect to \( x \) or \( y \)).

Cross sections \( \Rightarrow \) w.r.t \( y \)

\[
\text{larger radius} = \sqrt{2y} \quad \text{smaller radius} = y
\]

\[
\int_0^2 \pi (\sqrt{2y})^2 - \pi (y)^2 \, dy = \pi \int_0^2 2y - y^2 \, dy
\]

(d) (2 points) Set up but **do not solve** the integral for the volume of the solid obtained by rotating the region about the line \( x = 3 \) (you may integrate with respect to \( x \) or \( y \)).

Cross sections \( \Rightarrow \) w.r.t \( y \)

\[
\text{larger radius} = 3 - y \quad \text{smaller radius} = 3 - \sqrt{2y}
\]

\[
\int_0^2 \pi (3-y)^2 - \pi (3-\sqrt{2y})^2 \, dy = \pi \int_0^2 (3-y)^2 - (3-\sqrt{2y})^2 \, dy
\]

Shells \( \Rightarrow \) w.r.t \( x \)

\[
\text{radius} = x \quad \text{height} = x - \frac{x^2}{2}
\]

\[
\int_0^2 2\pi (x)(x - \frac{x^2}{2}) \, dx = 2\pi \int_0^2 x^2 - \frac{x^3}{2} \, dx
\]