Name: [Name]

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 15 minutes for this quiz.

1. (2 points) Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Find all numbers c that satisfy the conclusion.

\[ f(x) = \frac{1}{x} \text{ on } [1,3] \]

\[ 0 \]

\[ f(x) \text{ is continuous everywhere except } x=0, \text{ so } f(x) \text{ is continuous on } [1,3] \]

\[ 2 \]

\[ f(x) \text{ is differentiable everywhere except } x=0, \text{ so } f(x) \text{ is differentiable on } (1,3) \]

We have satisfied both hypotheses, so the MVT tells us there exists a \( c \) in \((1,3)\) such that

\[ f'(c) = \frac{f(3) - f(1)}{3-1} \]

\[ \text{want } f'(c) = \frac{\frac{1}{3} - 1}{2} = -\frac{1}{6} \]

\[ f'(c) = -\frac{1}{c^2} \]

\[ -\frac{1}{3} = -\frac{1}{c^2} \Rightarrow c^2 = 3 \]

\[ c = \pm \sqrt{3} \]

Only \( \sqrt{3} \) is in \((1,3)\). Therefore when \( c = \sqrt{3} \)

The conclusion of the MVT is satisfied.
2. Consider the region bounded by the curves, $y = 2x$ and $y = x^2$. Intersect at $x=0, x=2$

(a) (2 points) Set up but do not solve the integral for the volume of the solid obtained by rotating the region about the $x$-axis and integrating with respect to $x$.

Rotating around x-axis \( \Rightarrow \) cross sections perpendicular to x-axis \( \Rightarrow \) integrate with respect to $x$.

- Larger radius = $2x$
- Smaller radius = $x^2$

\[
\int_0^2 2\pi (2x)^2 - \pi (x^2)^2 \, dx = \pi \int_0^2 4x^2 - x^4 \, dx
\]

(b) (2 points) Set up but do not solve the integral for the volume of the solid obtained by rotating the region about the x-axis and integrating with respect to $y$.

Rotating around x-axis \( \Rightarrow \) shells parallel to x-axis \( \Rightarrow \) integrate with respect to $y$.

- Radius = $y$
- Height = $\sqrt{y} - \frac{y}{2}$

\[
\int_0^4 2\pi (y)(\sqrt{y} - \frac{y}{2}) \, dy = 2\pi \int_0^4 y^{3/2} - \frac{y^2}{2} \, dy
\]

(c) (2 points) Set up but do not solve the integral for the volume of the solid obtained by rotating the region about the $y$-axis (you may integrate with respect to $x$ or $y$).

Cross section \( \Rightarrow \) w.r.t. $y$

- Larger radius = $\sqrt{y}$
- Smaller radius = $y^2/2$

\[
\int_0^4 \pi (\sqrt{y})^2 - \pi (\frac{y^2}{2})^2 \, dy = \pi \int_0^4 y - \frac{y^2}{4} \, dy
\]

(d) (2 points) Set up but do not solve the integral for the volume of the solid obtained by rotating the region about the line $x = 3$ (you may integrate with respect to $x$ or $y$).

Cross section \( \Rightarrow \) w.r.t. $x$

- Larger radius = $3 - y^2/2$
- Smaller radius = $3 - \sqrt{y}$

\[
\int_0^4 \pi (3 - \frac{y^2}{2})^2 - \pi (3 - \sqrt{y})^2 \, dy = \pi \int_0^4 (3 - \frac{y^2}{2})^2 - (3 - \sqrt{y})^2 \, dy
\]