You have 15 minutes for this quiz – no calculators allowed.

1. (2 points) Explain carefully why \( f(x) = 3x^5 + x^3 + x - 15 \) cannot have two real roots.

   **Proof:** Suppose \( f(x) = 3x^5 + x^3 + x - 15 \) has two roots \( a \) and \( b \).
   
   Then \( f(a) = 0, f(b) = 0 \).
   
   \( f \) polynomial \( \Rightarrow f \) continuous on \([a,b]\)
   
   and \( f \) differentiable on \((a,b)\)
   
   Then by **Rolle's Theorem**, \( \exists c \) in \((a,b)\) such that \( f'(c) = 0 \).
   
   \[ f'(x) = 15x^4 + x^2 + 1 \]
   
   \[ f'(c) = 15c^4 + c^2 + 1 \geq 1 \]
   
   Hence \( f'(c) \neq 0 \).
   
   Hence \( f \) cannot have two real roots.

2. (2 points) Set up, but do not evaluate, the integral expressing the volume of a solid \( S \)
   
   where the base of \( S \) is the region enclosed by \( y = 4 - x^2 \) and the \( x \)-axis. Cross sections
   
   perpendicular to the \( x \)-axis are squares.

   \[
   \text{Volume} = \int_{-2}^{2} (4-x^2)^2 \, dx
   \]
3. (3 points each) Let \( R \) be the region bounded by \( y = 2 \) and the graph of \( y = 2 + \cos x \) on the interval \([\frac{\pi}{2}, \frac{3\pi}{2}]\). Set up, but do not evaluate, definite integrals which represent the given quantities. Use proper notation.

(a) The volume of the solid obtained when \( R \) is revolved around the \( y \)-axis.

\[
\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 2\pi \left(2 - (2 + \cos x)\right) \, dx.
\]

(b) The volume of the solid obtained when \( R \) is revolved around the line \( y = 4 \).

Outer radius = \( 4 - (2 + \cos x) \)

Inner radius = 2

\[
\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \pi \left[\left(4 - (2 + \cos x)^2\right) - 2^2\right] \, dx.
\]