1. (2 points) Show that the equation \(3x + 2\cos x + 5 = 0\) has at most one real root.

Let \(p(x) = 3x + 2\cos x + 5\) have 2 distinct real roots, say \(x_1\) and \(x_2\), i.e. \(p(x_1) = p(x_2) = 0\). Since \(p(x)\) is continuous on \([x_1, x_2]\), differentiable on \((x_1, x_2)\), Rolle's theorem applies. i.e. \(\exists c \in (x_1, x_2) \text{ s.t. } f'(c) = 0\). But \(f'(x) = 3 - 2\sin x\).
\[f'(c) = 3 - 2\sin c = 0 \implies \sin c = \frac{3}{2} > 1 \quad \text{not possible.}\]

2. (2 points) For \(f(x) = 5x^2 + 3x + 7\). Find \(c\) which satisfies the conclusion of the Mean Value Theorem on the interval \([0, 3]\).

By Mean Value theorem, \(f(3) - f(0) = (3-0) f'(c) \text{ for some } c \in (0, 3)\).
\[
\begin{align*}
\Rightarrow & \quad 5 \cdot 9 + 3 \cdot 3 + 7 - 7 = 3(10c + 3) \\
\Leftrightarrow & \quad 45 + 9 = 30c + 9 \\
\Rightarrow & \quad c = \frac{45}{30} = \frac{3}{2} \in (0, 3).
\end{align*}
\]
3. (a) Points given: Let $R$ be the region bounded by the $x$-axis and the graph of $y = \cos(x) + 3$ on the interval $[0, 2\pi]$. Set up, but do not evaluate, definite integrals which represent the

\[ V = \int_0^{2\pi} \pi x^2 \, dx, \quad x = \cos x + 3. \]

\[ = \int_0^{2\pi} \pi (\cos x + 3)^2 \, dx. \]

(b) The volume of the solid obtained when $R$ is revolved around the line $y = 19$.

\[ V = \int_0^{2\pi} \left[ \pi (19)^2 - \pi \left( 19 - (\cos x + 3) \right) \right] \, dx \]

(c) The volume of the solid with base $R$ for which the cross-sections perpendicular to the $x$-axis are semi-circles.

Radius of the semi-circle cross section is $\frac{1}{2} (y) = \frac{1}{2} (\cos x + 3)$.

\[ \therefore \quad \text{Area} = \frac{1}{2} \pi \left( \frac{\cos x + 3}{2} \right)^2 \]

\[ \therefore \quad V = \int_0^{2\pi} \frac{1}{2} \pi \left( \frac{\cos x + 3}{2} \right)^2 \, dx. \]