Name

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 15 minutes for this quiz.

1. (2.5 pts) Find the definite integral.

\[
\int_{\sqrt{\frac{\pi}{2}} - 1}^{\sqrt{\pi} - 1} x \sin(x^2 + 1) \, dx
\]

Let \( u = x^2 + 1 \)

\[
\frac{du}{dx} = 2x
\]

\[
\frac{du}{2} = x \, dx
\]

If \( x = \sqrt{\frac{\pi}{2}} - 1 \) \( \Rightarrow \) \( u = \frac{\pi}{2} \)

If \( x = \sqrt{\pi} - 1 \) \( \Rightarrow \) \( u = \pi \)

\[
= \int_{\frac{\pi}{2}}^{\pi} \frac{\sin(u)}{2} \, du = -\frac{\cos(u)}{2} \bigg|_{\frac{\pi}{2}}^{\pi}
\]

\[
= -\frac{1}{2} \left( \cos(\pi) - \cos\left(\frac{\pi}{2}\right) \right)
\]

\[
= -\frac{1}{2} (-1 - 0) = \frac{1}{2}
\]

2. (2.5 pts) Find the indefinite integral

\[
\int \frac{(\ln(3x))^2}{x} \, dx
\]

Let \( u = \ln(3x) \)

\[
\frac{du}{dx} = \frac{1}{3x} \cdot 3
\]

\[
du = \frac{dx}{x}
\]

\[
= \int u^2 \, du = \frac{u^3}{3} + C
\]

\[
= \frac{[\ln(3x)]^3}{3} + C
\]
3. Consider the region in the first quadrant of the plane bounded by the curves \( y = 9x \) and 
\( y = x^3 \). In the following manner set up, but do not evaluate, integrals which represent the 
area of this region.

(a) (2 pts) Integrate with respect to \( x \).

\[
q \cdot x = x^3, \quad 0 \leq x^3 - 9x, \quad 0 = x(x^2 - 9) \quad x = 0, \quad x = \pm 3
\]

Take \( x = 0, x = 3 \) because region is in the 1st quadrant.

\[
A = \int_{0}^{3} (q \cdot x - x^3) \, dx
\]

(b) (2 pts) Integrate with respect to \( y \).

\[
\begin{align*}
&\text{If} \quad x = 0 \Rightarrow y = 0 \\
&\text{If} \quad x = 3 \Rightarrow y = 27 \\
&y = x^3 \Rightarrow 3\sqrt[3]{y^3} = x \\
y = x^3 \Rightarrow 3\sqrt[3]{y} = x \\
A = \int_{0}^{27} (3\sqrt[3]{y^3} - \frac{y}{q}) \, dy
\end{align*}
\]

(c) (1 pt) Determine the area of the region by completing any necessary integration.

\[
A = \int_{0}^{3} (q \cdot x - x^3) \, dx = \left[ \frac{9x^2}{2} - \frac{x^4}{4} \right]_{0}^{3}
\]

\[
= \frac{9 \cdot 3^2}{2} - \frac{3^4}{4} = \frac{3^4}{2} - \frac{3^4}{4} = \frac{3^4}{4} = \frac{81}{4}
\]