1. (3 points) Evaluate
\[ \int \csc^2(4 \sin(4\theta)) \cos(4\theta) \, d\theta \]
Let \( u = 4 \sin(4\theta) \). Then \( du = 16 \cos(4\theta)d\theta \), so \( du/16 = \cos(4\theta)d\theta \). Therefore,
\[ \int \csc^2(4 \sin(4\theta)) \cos(4\theta) \, d\theta = \int \csc^2(u)/16 du = -\cot(u)/16 + C = -\cot(4 \sin(4\theta))/16 + C \]

2. (3 points) Evaluate
\[ \int_1^2 \frac{e^{-6/x}}{x^2} \, dx \]
Let \( u = -6/x \), \( du = 6/x^2 dx \), \( du/6 = 1/x^2 dx \), \( u(1) = -6 \), and \( u(2) = -3 \). Then
\[ \int_{-6}^{-3} e^u du/6 = e^u \bigg|_{-3}^{-6} = \frac{e^{-3}}{6} - \frac{e^{-6}}{6} \]

3. Consider the region \( R \) bounded by \( y = 1 \), \( y = 2 \), and \( y = x^2 \). Set up, but do not evaluate, definite integrals which represent the area of \( R \), in each of the following manners. Use proper notation. Your answers to (a) and (b) should have different integrands.

(a) (2 points) Integrate with respect to \( x \).
A picture might help:

The intersection points occur at when \( x^2 = 1 \) (so \( x = \pm 1 \)) and \( x^2 = 2 \) (so \( x = \pm \sqrt{2} \)).
From \( x = -\sqrt{2} \) to \( x = -1 \), the top curve is \( y = 2 \) and the bottom curve is \( y = x^2 \).
From \( x = -1 \) to \( x = 1 \), the top curve is \( y = 2 \) and the bottom curve is \( y = 1 \). From \( x = 1 \) to \( x = \sqrt{2} \), the top curve is \( y = 2 \) and the bottom curve is \( y = x^2 \). Thus, we have
\[ \int_{-\sqrt{2}}^{-1} (2 - x^2) \, dx + \int_{-1}^{1} (2 - 1) \, dx + \int_{1}^{\sqrt{2}} (2 - x^2) \, dx \]
Alternatively, if you notice that the region is symmetric about the \( y \)-axis, you could just find the area from 0 to \( \sqrt{2} \) and double it: \( 2 \int_{0}^{1} (2 - 1) \, dx + 2 \int_{1}^{\sqrt{2}} (2 - x^2) \, dx \).

(b) (2 points) Integrate with respect to \( y \).
We need to reframe everything in terms of \( y \). So \( y = x^2 \) becomes \( x = \pm \sqrt{y} \) (\( x = \sqrt{y} \) is the curve in the first quadrant above and \( x = -\sqrt{y} \) is the curve in the second quadrant). The right curve is \( x = \sqrt{y} \) while the left curve is \( x = -\sqrt{y} \), so we get
\[ \int_{1}^{2} (\sqrt{y} - (-\sqrt{y}) \, dy = \int_{1}^{2} 2 \sqrt{y} \, dy \]
Alternatively, note that the picture is symmetric about the \( y \)-axis, so we could find \( \int_{1}^{2} \sqrt{y} \, dy \) and double it.