Name: Yeli

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 15 minutes for this quiz.

1. (3 points) Evaluate the definite integral.

\[
\int_0^1 \frac{e^x}{1 + e^{2x}} \, dx = \int_{e^0}^{e^1} \frac{\sqrt{u}}{1+u^2} \, du
\]

\[u = e^x\]
\[du = e^x \, dx\]
\[dx = \frac{du}{e^x} = \frac{du}{u}\]

\[= \tan^{-1}(u) \bigg|_1^e\]
\[= \tan^{-1}(e) - \tan^{-1}(1)\]
\[= \tan^{-1}(e) - \frac{\pi}{4}\]

2. (3 points) Evaluate the indefinite integral.

\[
\int \frac{\cos x}{\sqrt{1 + \sin x}} \, dx = \int \frac{\cos x}{\sqrt{u}} \cdot \frac{du}{\cos x}
\]

\[u = 1 + \sin x\]
\[du = \cos x \, dx\]
\[dx = \frac{du}{\cos x}\]

\[= 2u^{\frac{3}{2}} + C\]
\[= 2(1 + \sin x)^{\frac{3}{2}} + C\]
\[= 2\sqrt{1 + \sin x} + C\]
3. (4 points) Consider the region bounded by the curves \( y = \sin x \) and \( y = \frac{2x}{\pi} \) (Hint this line goes through the point \( \left( \frac{\pi}{2}, 1 \right) \))

(a) Sketch the region.

(b) Set up the integral for the area of this region, integrating with respect to \( x \). Do not evaluate

\[
\int_{-\pi/2}^{0} \frac{2x}{\pi} - \sin x \, dx + \int_{0}^{\pi/2} \sin x - \frac{2x}{\pi} \, dx
\]

(c) Set up the integral for the area of this region, integrating with respect to \( y \). Do not evaluate

\[
\int_{-\frac{\pi}{2}}^{0} \sin^{-1}(y) - \frac{\pi y}{2} \, dy + \int_{0}^{1} \frac{\pi y}{2} - \sin^{-1}(y) \, dy
\]

(d) Find the area of the region by solving one of the integrals you have set up.

\[
\cos \text{er to integrate w.r.t. } x
\]

\[
\int_{-\pi/2}^{0} \frac{2x}{\pi} - \sin x \, dx + \int_{0}^{\pi/2} \sin x - \frac{2x}{\pi} \, dx
\]

\[
= \frac{x^2}{\pi} + \cos x \bigg|_{-\pi/2}^{0} - \cos x - \frac{x^2}{\pi} \bigg|_{0}^{\pi/2}
\]

\[
= \left( \frac{\pi}{4} + \left( \frac{\pi}{2} \right)^2 \right) + \left( - \cos \frac{\pi}{2} - \frac{(\pi/2)^2}{\pi} \right) - \left( - \cos 0 - \frac{0^2}{\pi} \right)
\]

\[
= 1 - \frac{\pi}{4} - 0 + 0 - \frac{\pi}{4} + 1 + 0
\]

\[
= 2 - \frac{2\pi}{4} = 2 - \frac{\pi}{2}
\]