Math 220 (Section ADF)  Quiz 10  Spring 2012

Name

You have 15 minutes for this quiz – no calculators allowed.

1. (2 points) Evaluate the integral \( \int \cos x \sin (\sin x) \, dx \).

   Let \( u = \sin x \)
   \[ du = \cos x \, dx \]

   \[ \int \cos x \sin (\sin x) \, dx = \int \sin u \, du = -\cos u + c \]
   \[ = -\cos (\sin x) + c \]

2. (3 points) Evaluate the integral \( \int_1^2 \frac{7x^11}{x^4+2} \, dx \).

   Let \( u = x^4 + 2 \) \( \to \)
   \[ du = 4x^3 \, dx \]
   \[ \frac{du}{4} = x^3 \, dx \]

   Note that \( \int_1^2 \frac{7x^{11}}{x^4+2} \, dx = \int_1^2 \frac{7 \cdot 8 \cdot x^3}{x^4+2} \, dx \)

   By (1), \( x^4 = u - 2 \)
   \[ \Rightarrow x^8 = (u - 2)^2 \]

   When \( x = 1 \), \( u = 1^4 + 2 = 1 + 2 = 3 \)
   When \( x = 2 \), \( u = 2^4 + 2 = 16 + 2 = 18 \)

   \[ \int_1^2 \frac{7 \cdot 8 \cdot x^3}{x^4+2} \, dx = \int_3^9 \frac{7(u-2)^2}{4u} \, du = \frac{7}{4} \int_3^9 \frac{u^2 + 4u - 4u}{u} \, du = \frac{7}{4} \int_3^9 (u + \frac{4}{u} - 4) \, du \]

   \[ \int_3^9 (u + \frac{4}{u} - 4) \, du = \left. \frac{7}{4} \left( u^2 + 4\ln u - 4u \right) \right|_3^9 = \frac{7}{4} \left( 9^2 - 3^2 - 4\ln 3 + 4(3) \right) \]

   \[ = \frac{7}{4} \left( (18)^2 + 4\ln 18 - 4(18) - \frac{3^2}{2} - 4\ln 3 + 4(3) \right) \]
3. (1 point) Evaluate \( \int_{-4}^{4} \frac{t^3}{2t^6 + t^6} \, dt \)

\[
\frac{t^3}{2t^6 + t^6} \text{ is an odd function!}
\]

Hence \( \int_{-4}^{4} \frac{t^3}{2t^6 + t^6} \, dt = 0 \)

4. (4 points) Consider the finite region bounded by the curve \( y = 2e^x \) and the lines \( y = 4 \) and \( y = 6 \). In the following manner set up, but do not evaluate, definite integrals which represent the area of this region.

\[
\begin{align*}
&y = 6 \quad \text{when } y = 4, \quad x = \ln \left( \frac{4}{2} \right) = \ln 2 \\
&y = 2e^x \quad \text{when } y = 6, \quad x = \ln \left( \frac{6}{2} \right) = \ln 3 \\
&y = e^x
\end{align*}
\]

(a) Integrate with respect to \( x \):

\[
\int_{0}^{\ln 2} (6 - 4) \, dx + \int_{\ln 2}^{\ln 3} (6 - 2e^x) \, dx
\]

(b) Integrate with respect to \( y \). (The integrands in parts (a) and (b) should be different.)

\[
\int_{4}^{6} 2e^x \ln \left( \frac{y}{2} \right) \, dy
\]