QUIZ 1 BD6

Duration: 25 minutes

**Question 1.** (7 pts) Using the formula \( \sin(x + y) = \sin x \cos y + \sin y \cos x \) show that

\[
\sin a + \sin b = 2 \sin\left(\frac{a + b}{2}\right) \cos\left(\frac{a - b}{2}\right).
\]

**Solution.** We have by just plugging in \(-y\) instead of \(y\) and noticing \(\sin(-y) = -\sin y\),

\[
\sin(x - y) = \sin x \cos y - \sin y \cos x.
\]

Thus summing these up gives

\[
\sin(x + y) + \sin(x - y) = 2 \sin x \cos y.
\]

Now just set \(a = x + y, b = x - y\). Then \(x = \frac{a+b}{2}, y = \frac{a-b}{2}\) which shows what we desired.

**Question 2.** (3 pts) Find the equation of the parabola passing through the points \((1, 0), (2, -2), (3, 0)\).

**Solution.** First and the third of these points gives the \(x\)-intercepts of the parabola, so equation must be of the form \(a(x - 1)(x - 3)\). To find \(a\) just plug in the second point

\[
-2 = a(2 - 1)(2 - 3)
\]

Thus \(a = 2\), and the answer is \(y = 2(x - 1)(x - 3)\).

Another way of solving this is to plug in the points into the equation of a generic polynomial \(y = ax^2 + bx + c\). In this way we find three equations in three variables, which can easily be solved.