Name: KEY

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 15 minutes for this quiz.

1. (2 pts) There is an angle $\theta$ for which $\sin \theta = \frac{\sqrt{3}}{2}$, $\cos \theta = -\frac{1}{2}$ and $\tan \theta = -\sqrt{3}$. What is the value of $\cos(\pi - \theta)$?

   $\cos(\pi - \theta) = \frac{1}{2}$

   \[ \cos(\pi - \theta) = \cos \pi \cos \theta + \sin \pi \sin \theta \]
   \[ = (-1)(\cos \theta) + 0 \]
   \[ = -\cos \theta \]
   \[ = -\left( -\frac{1}{2} \right) \]
   \[ = \frac{1}{2}. \]

2. Suppose that $h(x) = \frac{1}{\sqrt{(x^2 - 2)^3}}$.

   (a) (2 pts) Find a function $f$ of the form $f(x) = x^k$, $k \neq 1$, and a function $g$ such that $(f \circ g)(x) = h(x)$.

   $\overline{f(x)} = x^{-\frac{3}{2}}$

   $g(x) = x^2 - 2$

   (b) (1 pt) Find the domain of $h(x)$.

   $5\sqrt{(x^2 - 2)^3} \neq 0 \Rightarrow x^2 - 2 \neq 0$

   $\Rightarrow x \neq \pm \sqrt{2}$

   $\mathbb{D} = (-\infty, -\sqrt{2}) \cup (-\sqrt{2}, \sqrt{2}) \cup (\sqrt{2}, \infty)$
3. (2 pts) Determine real numbers \( a \) and \( b \) so that the expression \( 3 \tan^2 \theta + 7 \sec^2 \theta \) can be rewritten as \( a \sec^2 \theta + b \).

\[
3 \tan^2 \theta + 7 \sec^2 \theta \\
= 3(\sec^2 \theta - 1) + 7 \sec^2 \theta \\
= 3 \sec^2 \theta - 3 + 7 \sec^2 \theta \\
= 10 \sec^2 \theta - 3
\]

\[
\begin{align*}
\text{if} \quad a &= 10 \\
\text{if} \quad b &= -3
\end{align*}
\]

4. (1 pt) Given that \( f(x) \) is an even function and that \( g(x) = -3f(x) - 4 \), is \( g(x) \) an odd function, an even function or neither? Give a clear justification for your answer.

\[g(-x) = -3f(-x) - 4\]
\[= -3f(x) - 4 \quad \text{since } f \text{ is even}\]
\[= g(x) \]

\[
\text{even}
\]

5. (2 pts) Carefully sketch the graph of \( y = 5 + 2^{-x} \). If there are any horizontal or vertical intercepts or asymptotes, then their locations should be accurately shown on your graph.

\[
y = 2^x
\]

Flip over \( y \)-axis \((y = 2^{-x})\)
Move up \( \leq (y = 5 + 2^{-x})\)

\[
\begin{array}{c}
y = 5 \\
\text{asymptote}
\end{array}
\]

\[
(y, 0) \quad \text{(0, 6)}
\]
Name **KEY**

- No calculators allowed.
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1. (2 pts) There is an angle \( \theta \) for which \( \sin \theta = \frac{\sqrt{3}}{2} \), \( \cos \theta = -\frac{1}{2} \) and \( \tan \theta = -\sqrt{3} \). What is the value of \( \sin(\pi - \theta) \)?

\[
\sin(\pi - \theta) = \frac{\sqrt{3}}{2}
\]

or:
\[
\sin(\pi - \theta) = \sin \pi \cos \theta - \cos \pi \sin \theta
\]
\[
= 0 - (-1) \sin \theta
\]
\[
= \frac{\sqrt{3}}{2}
\]

2. Suppose that \( h(x) = \frac{1}{\sqrt{x^2 - 5}} \).

(a) (2 pts) Find a function \( f \) of the form \( f(x) = x^k \), \( k \neq 1 \), and a function \( g \) such that \((f \circ g)(x) = h(x)\).

\[
f(x) = x^{-\frac{3}{4}}
\]
\[
g(x) = x^2 - 5
\]

(b) (1 pt) Find the domain of \( h(x) \).

\[
\sqrt{x^2 - 5} \neq 0 \Rightarrow x^2 - 5 \neq 0
\]
\[
\Rightarrow x \neq \pm \sqrt{5}
\]
\[
(-\infty, -\sqrt{5}) \cup (-\sqrt{5}, \sqrt{5}) \cup (\sqrt{5}, \infty)
\]
3. (2 pts) Determine real numbers $a$ and $b$ so that the expression $3 \tan^2 \theta + 7 \sec^2 \theta$ can be rewritten as $a \tan^2 \theta + b$.

\[
\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}
\]

\[
\tan^2 \theta + 1 = \sec^2 \theta
\]

\[
3 \tan^2 \theta + 7 (\tan^2 \theta + 1)
\]

\[
= 3 \tan^2 \theta + 7 \tan^2 \theta + 7
\]

\[
= 10 \tan^2 \theta + 7
\]

\[
\begin{align*}
a &= 10 \\
b &= 7
\end{align*}
\]

4. (1 pt) Given that $f(x)$ is an odd function and that $g(x) = -3f(x) - 4$, is $g(x)$ an odd function, an even function or neither? Give a clear justification for your answer.

\[
g(-x) = -3f(-x) - 4
\]

\[
= -3(-f(x)) - 4 \quad \text{since } f \text{ is odd}
\]

\[
= 3f(x) - 4.
\]

(neither)

5. (2 pts) Carefully sketch the graph of $y = -3 - 2^x$. If there are any horizontal or vertical intercepts or asymptotes, then their locations should be accurately shown on your graph.

$y = 2^x$.

Flip over x-axis, $(y = -2^x)$
move down 3. $(y = -3 - 2^x)$

\[
\begin{align*}
&{\text{y}} = -3 \quad \text{asymptote} \\
&{(0, 4)}
\end{align*}
\]

$(1, -5)$