Name __________________________

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 15 minutes to complete this quiz.

1. (3 points) Let \( f(x) = \frac{1}{1 + \sqrt{2 - x^2}} \) and \( g(x) = x^2 - 2 \). What is \((g \circ f)(x)\)? Find the domains of \( f, g, g \circ f \).

\[
(g \circ f)(x) = g\left(f(x)\right) = g\left(\frac{1}{1 + \sqrt{2 - x^2}}\right) = \left(\frac{1}{1 + \sqrt{2 - x^2}}\right)^2 - 2.
\]

**Domain of \( f \):** Notice that the denominator \( 1 + \sqrt{2 - x^2} \) is never zero and \( \sqrt{2 - x^2} \) is defined if \( 2 - x^2 \geq 0 \), i.e. \( x \in [-\sqrt{2}, \sqrt{2}] \).

\( \therefore \) Domain of \( f \) is \([-\sqrt{2}, \sqrt{2}]\).

**Domain of \( g \):** \( g(x) = x^2 - 2 \) is a polynomial which is defined for all real values \( x \). i.e. Domain of \( g \) is \( \mathbb{R} \).

**Domain of \( g \circ f \):** Again, the denominator is never zero and \( \sqrt{2 - x^2} \) makes sense iff \( 2 - x^2 \geq 0 \), i.e. \( 2 \geq x^2 \) i.e. \( \sqrt{2} \geq |x| \).

\( \therefore \) Domain of \( g \circ f \) is \( x \in [-\sqrt{2}, \sqrt{2}] \).

2. (2 points) Evaluate \( \csc^2\left(\frac{7\pi}{6}\right) - \cot^2\left(\frac{7\pi}{6}\right) \).

**Method I**

\[
\csc^2\left(\frac{7\pi}{6}\right) - \cot^2\left(\frac{7\pi}{6}\right) = \left[1 + \cot^2\left(\frac{7\pi}{6}\right)\right] - \cot^2\left(\frac{7\pi}{6}\right) = 1
\]

(using the identity \( 1 + \cot^2 x = \csc^2 x \)).

**Method II**

\[
\csc^2\left(\frac{7\pi}{6}\right) - \cot^2\left(\frac{7\pi}{6}\right) = \frac{1}{\sin^2\left(\frac{7\pi}{6}\right)} - \frac{\cos^2\left(\frac{7\pi}{6}\right)}{\sin^2\left(\frac{7\pi}{6}\right)}
\]

\[
= \frac{1 - \cos^2\left(\frac{7\pi}{6}\right)}{\sin^2\left(\frac{7\pi}{6}\right)} = \frac{\sin^2\left(\frac{7\pi}{6}\right)}{\sin^2\left(\frac{7\pi}{6}\right)} = 1
\]

\( \therefore 1 - \cos^2 x = \sin^2 x \)
3. (2 points) Find the function \( f(x) = k a^x \) such that the points \((1, 2)\) and \((-1, 2)\) lie on the graph of \( f \) given that the function \( f(x) \) is defined for all real numbers.

\[
f(x) = k a^x, \quad (1, 2) \text{ and } (-1, 2) \text{ lie on the graph of } f, \text{ means}
\]

\[
a = k a \quad \text{and} \quad 2 = k a^1 \implies 2 = \frac{k}{a}.
\]

**Method I**

\[
a = k a \quad \text{and} \quad 2 = \frac{k}{a} \implies k a = \frac{k}{a} \implies a^2 = 1 \implies a = \pm 1
\]

but \( a > 0 \), \( \therefore \ a = 1 \implies k = 2 \). Hence \( f(x) = 2 \).

**Method II**

\[
2 = k a \quad \text{and} \quad 2 = \frac{k}{a} \implies 2 = k a \quad \text{and} \quad 2a = k
\]

i.e. \( 2 = (k a) a \implies a^2 = 1 \implies a = \pm 1 \) but \( a > 0 \).

\( \therefore a = 1 \implies k = 2 \). Hence \( f(x) = 2 \).

4. (2 points) Draw the graph of the function \( f(x) = 1 - 4|x - 2| \) on its domain.

Let's draw \( g(x) = |x| \) and then shift it to the right by 2 units to get the graph of \( y = |x - 2| \).

Now reflect it about \( x \)-axis to get the graph of \( y = -4|x - 2| \).

Now shift up by 1 unit to get the graph of \( f(x) = 1 - 4|x - 2| \).