Name S O L U T I O N S (circle your TA discussion section)

- AD1, TR 11:00-11:50, Amita Malik
- AD3, TR 1:00-1:50, Neha Gupta
- AD5, TR 2:00-2:50, Neha Gupta
- AD8, TR 1:00-2:50, Hannah Kolb-Spinoza
- BD1, TR 2:00-2:50, Stephen Longfield
- BD3, TR 11:00-11:50, Michael Santana
- BD5, TR 2:00-2:50, Stephen Berning
- BD7, TR 3:00-3:50, Stephen Berning
- AD2, TR 1:00-1:50, Amita Malik
- AD4, TR 1:00-1:50, Meghan Galiardi
- AD7, TR 3:00-3:50, Meghan Galiardi
- AD9, TR 9:00-10:50, Vicki Reuter
- BD2, TR 8:00-8:50, Eliana Duarte
- BD4, TR 9:00-9:50, Eliana Duarte
- BD6, TR 1:00-1:50, Faruk Temur
- BD8, TR 3:00-3:50, Stephen Longfield

- Sit in your assigned seat (circled below).
- Circle your TA discussion section.
- Do not open this test booklet until I say START.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- Remove hats and sunglasses.
- You must show sufficient work to justify each answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Do not leave early unless you are at the end of a row.
- Quit working and close this test booklet when I say STOP.
- Quickly turn in your test to me or a TA and show your Student ID.
1. (10 points) Find a formula for $f(x)$ given that $f''(x) = 3\sin x$, $f(0) = 15$ and $f'(0) = 2$.

$$f''(x) = 3\sin x \implies f'(x) = -3\cos x + C$$

$$2 = f'(0) = -3\cos(0) + C$$

$$C = 5$$

$$f'(x) = -3\cos x + 5 \implies f(x) = -3\sin x + 5x + D$$

$$15 = f(0) = -3\sin(0) + 5(0) + D$$

$$D = 15$$

$$f(x) = -3\sin x + 5x + 15$$

2. (10 points) Fill in the missing information for the following theorem.

**Mean Value Theorem** Let $f$ be a function that satisfies the following two hypotheses:

1. $f$ is **continuous** on the closed interval $[a, b]$.
2. $f$ is **differentiable** on the open interval $(a, b)$.

Then there is a number $c$ in $(a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$
3. (10 points) Evaluate the definite integral. Simplify your answer.

\[
\int_{-5}^{5} (7x^9 - 3x^5 + 4) \, dx = 7 \int_{-5}^{5} x^9 \, dx - 3 \int_{-5}^{5} x^5 \, dx + \int_{-5}^{5} 4 \, dx
\]

\[
= 7 \cdot 0 - 3 \cdot 0 + \left[ 4x \right]_{-5}^{5}
\]

\[
= 0 - 0 + 4 \cdot 5 - 4 \cdot (-5)
\]

\[
= 20 - (-20)
\]

\[
= 40
\]

we used that \( \int_{-a}^{a} f(x) \, dx = 0 \) for integrable odd functions such as \( x^9 \) or \( x^5 \).

4. (10 points) Evaluate the definite integral. Simplify your answer.

\[
\int_{0}^{4} \frac{10x}{\sqrt{x^2 + 9}} \, dx = \int_{9}^{25} \frac{5}{\sqrt{u}} \, du = \int_{9}^{25} 5u^{-1/2} \, du
\]

\[
= \left[ \frac{5u^{1/2}}{1/2} \right]_{9}^{25}
\]

\[
= \left[ 10\sqrt{u} \right]_{9}^{25}
\]

\[
= 10\sqrt{25} - 10\sqrt{9}
\]

\[
= 50 - 30
\]

\[
= 20
\]
5. (10 points) Evaluate the indefinite integral.

\[
\int \frac{e^{4x}}{1 + e^{4x}} \, dx = \int \frac{\frac{1}{4} du}{u} = \frac{1}{4} \int \frac{1}{u} \, du = \frac{1}{4} \ln |u| + C
\]

\[
u = 1 + e^{4x}
\]

\[
du = 4e^{4x} \, dx
\]

\[
\frac{1}{4} \, du = e^{4x} \, dx
\]

6. (10 points) Evaluate the indefinite integral.

\[
\int \frac{8 - 6 \cos^3 x}{2 \cos^2 x} \, dx = \int \left( \frac{8}{2 \cos^2 x} - \frac{6 \cos^3 x}{2 \cos^2 x} \right) \, dx
\]

\[
= \int (4 \sec^2 x - 3 \cos x) \, dx
\]

\[
= 4 \tan x - 3 \sin x + C
\]
7. (10 points) Evaluate the indefinite integral.

\[
\int \sin^5 x \, dx = \int \sin^4 x \, \sin x \, dx \\
= \int \sin^3 x \, \cos^2 x \, dx \\
= \int (1 - \cos^2 x)^2 \, \sin x \, dx \\
= \int (1 - u^2)^2 \, (-du) \\
= \int (-1 + 2u^2 - u^4) \, du \\
= -u + \frac{2}{3} u^3 - \frac{1}{5} u^5 + C \\
= -\cos x + \frac{2}{3} \cos^3 x - \frac{1}{5} \cos^5 x + C
\]

8. (10 points) Suppose \( f \) is a polynomial and the graph of \( y = f(x) \) goes through points \((0, 4), (1, 3), (2, 14), (3, 67)\) and \((4, 216)\). Evaluate the following quantities and simplify your answer.

(a) \( \int_1^2 f'(x) \, dx = f(2) - f(1) \) by Fundamental Theorem of Calculus

\[
= 14 - 3 \\
= 11
\]

(b) \( \int_1^2 f'(2x) \, dx \)

\[
U = 2x \\
dU = 2 \, dx \\
\frac{1}{2} \, dU = dx \\
x = 1 \Rightarrow U = 2 \cdot 1 = 2 \\
x = 2 \Rightarrow U = 2 \cdot 2 = 4
\]

\[
= \frac{1}{2} \left( f(4) - f(2) \right) \\
= \frac{1}{2} \left( 216 - 14 \right) \\
= \frac{1}{2} (202) \\
= 101
\]
9. (10 points) Let $R$ be the finite region in the first quadrant which is bounded by the graphs of $y = \sqrt{x}$ and $y = x - 6$. In the following manner set up, but do not evaluate, definite integrals which represent the volume of the solid obtained when $R$ is revolved around the vertical line $x = 15$. Use proper notation.

(a) Integrate with respect to $x$.

$$\text{Vol} = \int_0^6 2\pi (15-x) \sqrt{x} \, dx + \int_6^9 2\pi (15-x) (\sqrt{x} - (x-6)) \, dx$$

(b) Integrate with respect to $y$. (The integrands in parts (a) and (b) should be different.)

$$\text{Vol} = \int_0^3 \left( \pi (15-y)^2 - \pi (15-(y+6))^2 \right) \, dy$$

Cross-section
10. (10 points) The graphs of \( y = x^3 \) and \( y = 5 - x^2 \) intersect somewhere on the interval \([1, 2]\). To estimate the \( x \)-value for this point of intersection, begin with an initial estimate of \( x_1 = 1 \) and determine a second estimate \( x_2 \) by applying Newton’s Method to an appropriate function. Show all work and write your answer either in decimal form or as a simplified fraction.

\[
\begin{align*}
    x^3 &= 5 - x^2 \\
    x^3 + x^2 - 5 &= 0 \\

    \text{Apply Newton's Method to} \\
    f(x) &= x^3 + x^2 - 5 \quad \text{with initial estimate } x_1 = 1 \\
    f'(x) &= 3x^2 + 2x \\
    x_1 &= 1 \quad \text{and} \quad x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \\
    x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} \\
    &= 1 - \frac{f(1)}{f'(1)} \\
    &= 1 - \frac{1^3 + 1^2 - 5}{3 \cdot 1^2 + 2 \cdot 1} \\
    &= 1 - \frac{-3}{5} \\
    &= \frac{8}{5} = 1.6
\end{align*}
\]
Students – do not write on this page!

1. (10 points) ________________
2. (10 points) ________________
3. (10 points) ________________
4. (10 points) ________________
5. (10 points) ________________
6. (10 points) ________________
7. (10 points) ________________
8. (10 points) ________________
9. (10 points) ________________
10. (10 points) ________________

TOTAL (100 points) __________