MATH 220 Test 3 Spring 2012

Name SOLUTIONS UIN or NetID ____________

(circle your TA discussion section)

- ▷ **AD1**, TR 11:00-11:50, Amita Malik
- ▷ **AD2**, TR 1:00-1:50, Amita Malik
- ▷ **AD3**, TR 1:00-1:50, Neha Gupta
- ▷ **AD4**, TR 1:00-1:50, Meghan Galiardi
- ▷ **AD5**, TR 2:00-2:50, Neha Gupta
- ▷ **AD7**, TR 3:00-3:50, Meghan Galiardi
- ▷ **AD8**, TR 1:00-2:50, Hannah Kolb-Spinoza
- ▷ **AD9**, TR 9:00-10:50, Vicki Reuter
- ▷ **BD1**, TR 2:00-2:50, Stephen Longfield
- ▷ **BD2**, TR 8:00-8:50, Eliana Duarte
- ▷ **BD3**, TR 11:00-11:50, Michael Santana
- ▷ **BD4**, TR 9:00-9:50, Eliana Duarte
- ▷ **BD5**, TR 2:00-2:50, Stephen Berning
- ▷ **BD6**, TR 1:00-1:50, Faruk Temur
- ▷ **BD7**, TR 3:00-3:50, Stephen Berning
- ▷ **BD8**, TR 3:00-3:50, Stephen Longfield

- Sit in your assigned seat (circled below).
- Circle your TA discussion section.
- Do not open this test booklet until I say *START*.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- Remove hats and sunglasses.
- You must show sufficient work to justify each answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Do not leave early unless you are at the end of a row.
- Quit working and close this test booklet when I say *STOP*.
- Quickly turn in your test to me or a TA and show your Student ID.

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	240	241	242	243	244	245	246	•	247	248	249	250	251	252	253	254	255	•	256	257	258	259	260	261	262	
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FRONT OF ROOM – 314 Altgeld Hall

1. (10 points) Find a formula for $f(x)$ given that $f''(x) = 3\sin x$, $f(0) = 15$ and $f'(0) = 2$.

$f''(x) = 3\sin x \Rightarrow f'(x) = -3\cos x + C$

$2 = f'(0) = -3\cos(0) + C$

$2 = -3 + C$

$C = 5$

$f'(x) = -3\cos x + 5 \Rightarrow f(x) = -3\sin x + 5x + D$

$15 = f(0) = -3\sin(0) + 5(0) + D$

$15 = D$

$f(x) = -3\sin x + 5x + 15$

2. (10 points) Fill in the missing information for the following theorem.

Mean Value Theorem Let f be a function that satisfies the following two hypotheses:

(1) f is continuous on the closed interval $[a, b]$.

(2) f is differentiable on the open interval (a, b).

Then there is a number c in (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

3. (10 points) Evaluate the definite integral. Simplify your answer.

$$\int_{-5}^{5} (7x^9 - 3x^5 + 4)\, dx = 7\int_{-5}^{5} x^9 dx - 3\int_{-5}^{5} x^5 dx + \int_{-5}^{5} 4\, dx$$

$$= 7 \cdot 0 - 3 \cdot 0 + \Big[4x\Big]_{-5}^{5}$$

$$= 20 - (-20)$$

$$= 40$$

we used that $\int_{-a}^{a} f(x)\,dx = 0$

for integrable odd functions such as x^9 or x^5

4. (10 points) Evaluate the definite integral. Simplify your answer.

$$\int_0^4 \frac{10x}{\sqrt{x^2+9}}\, dx = \int_9^{25} \frac{5\,du}{\sqrt{u}} = \int_9^{25} 5u^{-1/2}\,du$$

$u = x^2 + 9$

$du = 2x\,dx$

$5du = 10x\,dx$

$x = 0 \Rightarrow u = 0^2 + 9 = 9$

$x = 4 \Rightarrow u = 4^2 + 9 = 25$

$$= \left[5\frac{u^{1/2}}{1/2}\right]_9^{25}$$

$$= \Big[10\sqrt{u}\Big]_9^{25}$$

$$= 10\sqrt{25} - 10\sqrt{9}$$

$$= 50 - 30$$

$$= 20$$

5. (10 points) Evaluate the indefinite integral.

$$\int \frac{e^{4x}}{1+e^{4x}}\,dx = \int \frac{\frac{1}{4}du}{u} = \frac{1}{4}\int \frac{1}{u}\,du$$

$u = 1+e^{4x}$

$du = 4e^{4x}dx$

$\frac{1}{4}du = e^{4x}dx$

$$= \frac{1}{4}\ln|u| + C$$

$$= \frac{1}{4}\ln(1+e^{4x}) + C$$

6. (10 points) Evaluate the indefinite integral.

$$\int \frac{8-6\cos^3 x}{2\cos^2 x}\,dx = \int \left(\frac{8}{2\cos^2 x} - \frac{6\cos^3 x}{2\cos^2 x}\right)dx$$

$$= \int (4\sec^2 x - 3\cos x)\,dx$$

$$= 4\tan x - 3\sin x + C$$

7. (10 points) Evaluate the indefinite integral.

$$\int \sin^5 x\,dx = \int \sin^4 x \sin x\,dx$$

$$= \int (\sin^2 x)^2 \sin x\,dx$$

$$= \int (1-\cos^2 x)^2 \sin x\,dx \quad \left\{ \begin{array}{l} u = \cos x \\ du = -\sin x\,dx \\ -du = \sin x\,dx \end{array} \right.$$

$$= \int (1-u^2)^2(-du)$$

$$= \int (-1+2u^2-u^4)\,du$$

$$= -u + \tfrac{2}{3}u^3 - \tfrac{1}{5}u^5 + C$$

$$= -\cos x + \tfrac{2}{3}\cos^3 x - \tfrac{1}{5}\cos^5 x + C$$

8. (10 points) Suppose f is a polynomial and the graph of $y = f(x)$ goes through points $(0,4)$, $(1,3)$, $(2,14)$, $(3,67)$ and $(4,216)$. Evaluate the following quantities and simplify your answer.

(a) $\int_1^2 f'(x)\,dx = f(2) - f(1)$ by Fund. thm. of calculus

$= 14 - 3$

$= 11$

(b) $\int_1^2 f'(2x)\,dx = \int_2^4 f'(u)\left(\tfrac{1}{2}du\right) = \tfrac{1}{2}\int_2^4 f'(u)\,du$

$u = 2x$

$du = 2dx$

$\tfrac{1}{2}du = dx$

$x = 1 \Rightarrow u = 2\cdot 1 = 2$

$x = 2 \Rightarrow u = 2\cdot 2 = 4$

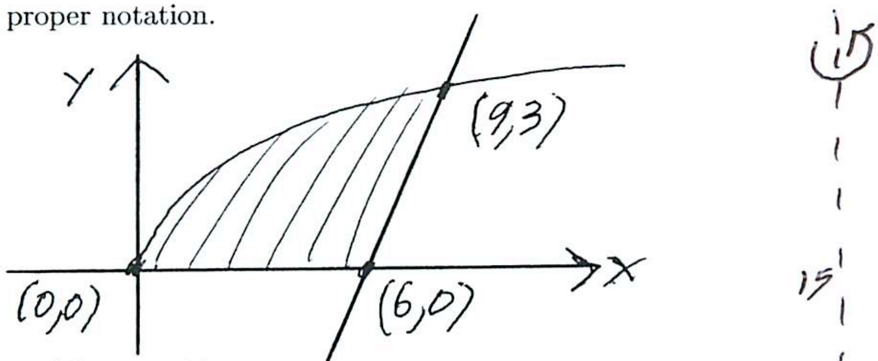
$= \tfrac{1}{2}\left(f(4) - f(2)\right)$ BY F.T.C.

$= \tfrac{1}{2}(216 - 14)$

$= \tfrac{1}{2}(202)$

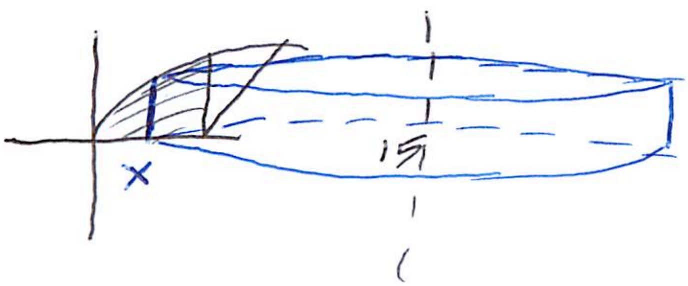
$= 10$

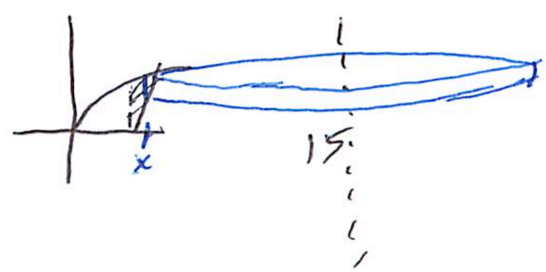
9. (10 points) Let **R** be the finite region in the first quadrant which is bounded by the graphs of $y = \sqrt{x}$ and $y = x - 6$. In the following manner set up, but do not evaluate, definite integrals which represent the volume of the solid obtained when **R** is revolved around the vertical line $x = 15$. Use proper notation.



(a) Integrate with respect to x.

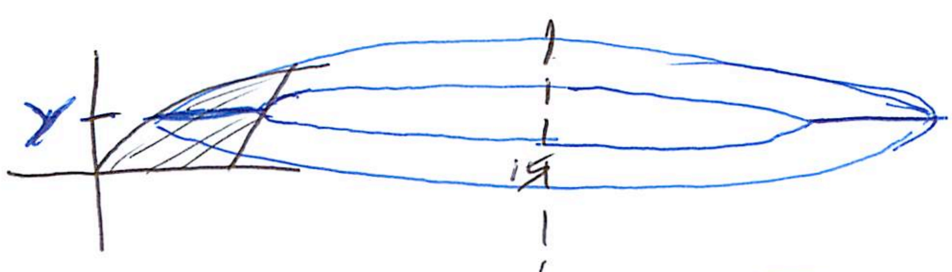
$$\text{VOL} = \int_0^6 2\pi \underbrace{(15-x)}_{\text{rad}} \underbrace{\sqrt{x}}_{\text{height}}\, dx + \int_6^9 2\pi \underbrace{(15-x)}_{\text{rad}} \underbrace{(\sqrt{x} - (x-6))}_{\text{height}}\, dx$$





(b) Integrate with respect to y. (The integrands in parts (a) and (b) should be different.)

$$\text{VOL} = \int_0^3 \left(\pi(\underbrace{15-y^2}_{\text{big radius}})^2 - \pi(\underbrace{15-(y+6)}_{\text{small radius}})^2\right) dy$$



cross-section

10. (10 points) The graphs of $y = x^3$ and $y = 5 - x^2$ intersect somewhere on the interval $[1, 2]$. To estimate the x-value for this point of intersection, begin with an initial estimate of $x_1 = 1$ and determine a second estimate x_2 by applying Newton's Method to an appropriate function. Show all work and write your answer either in decimal form or as a simplified fraction.

$$x^3 = 5 - x^2$$

$$x^3 + x^2 - 5 = 0$$

Apply Newton's Method to

$$f(x) = x^3 + x^2 - 5$$

with initial estimate $x_1 = 1$

$$f'(x) = 3x^2 + 2x$$

$x_1 = 1$ and $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ for $n \geq 1$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1 - \frac{f(1)}{f'(1)}$$

$$= 1 - \frac{1^3 + 1^2 - 5}{3 \cdot 1^2 + 2 \cdot 1}$$

$$= 1 - \frac{-3}{5}$$

$$= \frac{8}{5} = 1.6$$

Students – do not write on this page!

1. (10 points) ____________________

2. (10 points) ____________________

3. (10 points) ____________________

4. (10 points) ____________________

5. (10 points) ____________________

6. (10 points) ____________________

7. (10 points) ____________________

8. (10 points) ____________________

9. (10 points) ____________________

10. (10 points) ____________________

TOTAL (100 points) ____________