Math 220 – Test 3 Information

The test will be given during your lecture period on Wednesday (4-25-2012). No books, notes, scratch paper, calculators or other electronic devices are allowed. Bring a Student ID.

It may be helpful to look at

- [https://compass2g.illinois.edu/](https://compass2g.illinois.edu/) – homework solutions
- [http://www.math.illinois.edu/~murphyrf/teaching/M220/](http://www.math.illinois.edu/~murphyrf/teaching/M220/) – tests in my previous courses

- **Section 3.10 (Linear Approximation and Differentials)**
  - Be able to use a tangent line (or differentials) in order to approximate the value of a function near the point of tangency.

- **Section 4.2 (The Mean Value Theorem)**
  - Be able to precisely state *The Mean Value Theorem* and *Rolle’s Theorem*.
  - Be able to decide when functions satisfy the conditions of these theorems. If a function does satisfy the conditions, then be able to find the value of \( c \) guaranteed by the theorems.
  - Be able to use *The Mean Value Theorem*, *Rolle’s Theorem*, or earlier important theorems such as *The Intermediate Value Theorem* to prove some other fact. In the homework these often involved roots, solutions, \( x \)-intercepts, or intersection points.

- **Section 4.8 (Newton’s Method)**
  - Understand the graphical basis for Newton’s Method (that is, use the point where the tangent line crosses the \( x \)-axis as your next estimate for a root of a function).
  - Be able to apply Newton’s Method to approximate roots, solutions, \( x \)-intercepts, or intersection points.

- **Section 4.9 (Antiderivatives)**
  - Know antiderivative formulas for 0, \( k \) (a constant), \( \sin x \), \( \cos x \), \( \sec^2 x \), \( \csc^2 x \), \( \sec x \tan x \), \( \csc x \cot x \), \( e^x \), \( \frac{1}{1 + x^2} \), \( \frac{1}{\sqrt{1 - x^2}} \), \( x^n \) (\( n \neq -1 \)), \( x^{-1} = \frac{1}{x} \).
  - Be able to find general antiderivatives for functions which are sums or differences of constants multiplied by the above formulas (you may need to simplify first).
  - Be able to solve a differential equation where values for the function or its first or second derivative are given.
  - Be able to apply these rules to problems involving acceleration, velocity, or position.
  - You should know the acceleration due to gravity in terms of \( \text{ft/sec}^2 \) or \( \text{m/sec}^2 \).
• Section 5.1 (Areas and Distances)

– Use Riemann sums (left, right, or midpoint) to estimate area or total change in a quantity and state if your estimate is known to be an underestimate or overestimate. These sums will involve at most 8 subintervals.

– Use limits of Riemann sums to find the exact area or total change in a quantity. Being able to do this with right Riemann sums will be sufficient for this test.

– Understand sigma notation for sums and know that \[ \sum_{k=1}^{n} k = \frac{n(n+1)}{2}. \]

• Section 5.2 (The Definite Integral)

– Understand the definition of a definite integral as \( \int_{a}^{b} f(x) \, dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x^*_i) \Delta x \). Be able to more explicitly write out the appropriate limit for specific functions on given intervals. You may have to evaluate one such limit.

– Know the relationship between a definite integral and area. This should be understood regardless of whether or not the graph of the function being integrated is above or below the \( x \)-axis.

– Know the following properties of the definite integral.

* \( \int_{b}^{a} f(x) \, dx = - \int_{a}^{b} f(x) \, dx \)

* \( \int_{a}^{b} f(x) \, dx = 0 \)

* \( \int_{a}^{b} c \, dx = c(b - a) \) where \( c \) is any constant

* \( \int_{a}^{b} [f(x) + g(x)] \, dx = \int_{a}^{b} f(x) \, dx + \int_{a}^{b} g(x) \, dx \)

* \( \int_{a}^{b} cf(x) \, dx = c \int_{a}^{b} f(x) \, dx \) where \( c \) is any constant

* \( \int_{a}^{b} [f(x) - g(x)] \, dx = \int_{a}^{b} f(x) \, dx - \int_{a}^{b} g(x) \, dx \)

* \( \int_{a}^{c} f(x) \, dx + \int_{c}^{b} f(x) \, dx = \int_{a}^{b} f(x) \, dx \)

* If \( f(x) \geq 0 \) for \( a \leq x \leq b \), then \( \int_{a}^{b} f(x) \, dx \geq 0 \)

* If \( f(x) \geq g(x) \) for \( a \leq x \leq b \), then \( \int_{a}^{b} f(x) \, dx \geq \int_{a}^{b} g(x) \, dx \)

* If \( m \leq f(x) \leq M \) for \( a \leq x \leq b \), then \( m(b - a) \leq \int_{a}^{b} f(x) \, dx \leq M(b - a) \)
• **Section 5.3 (The Fundamental Theorem of Calculus)**
  – Be able to precisely state *The Fundamental Theorem of Calculus, Part 2*.
  – Be able to decide when functions satisfy the conditions of this theorem. If so, then be able to evaluate definite integrals with this theorem.

• **Section 5.4 (Indefinite Integrals and the Net Change Theorem)**
  – Know indefinite integral formulas for \( 0, k \) (a constant), \( \sin x \), \( \cos x \), \( \sec^2 x \), \( \csc^2 x \), \( \sec x \tan x \), \( \csc x \cot x \), \( e^x \), \( \frac{1}{1 + x^2} \), \( \frac{1}{\sqrt{1 - x^2}} \), \( x^n \) (\( n \neq -1 \)), \( x^{-1} = \frac{1}{x} \).
  – Know that the definite integral of a rate of change gives the total change. Be able to use this *Net Change Theorem* for applied problems involving rates of change such as velocity, acceleration, growth rates, etc.

• **Section 5.5 (The Substitution Rule)**
  – Be able to solve a wide variety of definite or indefinite integrals using substitution.
  – Be able to more quickly evaluate definite integrals on the interval \([-a, a]\) given that the integrand is continuous and either even or odd on that interval.

• **Section 6.1 (Areas between Curves)**
  – Be able to find areas between curves. This may require breaking the area up into the sum of two or more definite integrals.
  – Be able to integrate with respect to \( x \) or with respect to \( y \) to determine these areas.

• **Section 6.2 (Volumes)**
  – Be able to find volumes for solids formed by revolving a region around any vertical or horizontal line.
  – Be able to find volumes for solids formed by building upon some base and having cross-sections which are rectangles, squares, triangles, semi-circles, etc.
  – Be able to integrate with respect to \( x \) or with respect to \( y \) to determine these volumes.

• **Section 6.3 (Volumes by Cylindrical Shells)**
  – Be able to find volumes for solids formed by revolving a region around any vertical or horizontal line.
  – Be able to integrate with respect to \( x \) or with respect to \( y \) to determine these volumes.
• Section 6.5 (Average Value of a Function)
  - Be able to find the average value of a function.
  - Know the graphical interpretation of the average value of a function.
  - Know The Mean Value Theorem for Integrals.

• Section 7.2 (Trigonometric Integrals)
  - Be able to use substitution to solve definite or indefinite integrals involving trigonometric functions.
  - Be able to use basic trigonometric definitions and identities to help evaluate these integrals. In particular be able to use

\[
\begin{align*}
* \tan x &= \frac{\sin x}{\cos x} \\
* \cot x &= \frac{\cos x}{\sin x} \\
* \sec x &= \frac{1}{\cos x} \\
* \csc x &= \frac{1}{\sin x} \\
* \sin^2 x + \cos^2 x &= 1 \\
* \tan^2 x + 1 &= \sec^2 x \\
* 1 + \cot^2 x &= \csc^2 x \\
* \sin (2x) &= 2 \sin x \cos x \\
* \cos (2x) &= \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x \\
* \cos^2 x &= \frac{1}{2} + \frac{1}{2} \cos (2x) \\
* \sin^2 x &= \frac{1}{2} - \frac{1}{2} \cos (2x)
\end{align*}
\]