MATH 220  Test 2  Spring 2012

Name ___________________________  UIN ___________________________

(circle your TA discussion section)

- AD1, TR 11:00-11:50, Amita Malik
- AD2, TR 1:00-1:50, Amita Malik
- AD3, TR 1:00-1:50, Neha Gupta
- AD4, TR 1:00-1:50, Meghan Galiardi
- AD5, TR 2:00-2:50, Neha Gupta
- AD7, TR 3:00-3:50, Meghan Galiardi
- AD8, TR 1:00-2:50, Hannah Kolb-Spinoza
- AD9, TR 9:00-10:50, Vicki Reuter
- BD1, TR 2:00-2:50, Stephen Longfield
- BD2, TR 8:00-8:50, Eliana Duarte
- BD3, TR 11:00-11:50, Michael Santana
- BD4, TR 9:00-9:50, Eliana Duarte
- BD5, TR 2:00-2:50, Stephen Berning
- BD6, TR 1:00-1:50, Faruk Temur
- BD7, TR 3:00-3:50, Stephen Berning
- BD8, TR 3:00-3:50, Stephen Longfield

- Sit in your assigned seat (circled below).
- Circle your TA discussion section.
- Do not open this test booklet until I say START.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- Remove hats and sunglasses.
- You must show sufficient work to justify each answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Do not leave early unless you are at the end of a row.
- Quit working and close this test booklet when I say STOP.
- Quickly turn in your test to me or a TA and show your Student ID.

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FRONT OF ROOM – 314 Altgeld Hall
1. (2 points each) Circle **true** if the given statement is always true. Otherwise circle **false**.

(a) Given a polynomial \( f(x) \), if \( f'(x) \) is increasing on an open interval then \( f \) is concave up on that interval.

**true or false?**

\[ f' \text{ is increasing } \Rightarrow \text{ its derivative } \quad f'' > 0 \]
\[ \Rightarrow f \text{ is concave up} \]

(b) If \( f''(a) = 0 \) then there is an inflection point at \( x = a \).

**true or false?**

\[ f \text{ must be continuous and switch concavity at } x = a \text{ in order to have an inflection point there.} \]

\[ \text{Note: If } f(x) = x^4 \text{ then } f''(x) = 12x^2 \text{ equals 0 at } x = 0 \text{ yet } f \text{ has no inflection point.} \]

(c) \( \lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} \)

**true or false?**

This is only guaranteed to be true under the condition of l'Hopital's Rule (see section 4.4). It is not true in general.

(d) The solution to the differential equation \( \frac{dq}{dr} = 2q \) is an exponential function.

**true or false?**

\[ \frac{dq}{dr} = 2q \text{ has solution } q = Ce^{2r} \]
2. (8 points) Find \( h'(x) \) given that \( h(x) = 5x^4 - \sqrt[3]{x} + \sec x + \ln x \)

\[
h'(x) = 20x^3 - \frac{1}{3}x^{-2/3} + \sec x \cdot \tan x + \frac{1}{x}
\]

3. (8 points) Find \( \frac{dw}{dt} \) given that \( w = 10t^5 \sin^{-1} t \)

\[
\frac{dw}{dt} = (10t^5)'(\sin^{-1} t) + (10t^5)(\sin^{-1} t)' \\
= 50t^4 \sin^{-1} t + 10t^5 \cdot \frac{1}{\sqrt{1-t^2}}
\]

4. (8 points) Find \( f'(x) \) given that \( f(x) = \frac{x^3}{x^5 + 4x + 2} \)

\[
f'(x) = \frac{(x^3)'(x^5 + 4x + 2) - (x^3)(x^5 + 4x + 2)'}{(x^5 + 4x + 2)^2} \\
= \frac{3x^2(x^5 + 4x + 2) - x^3(5x^4 + 4)}{(x^5 + 4x + 2)^2}
\]
5. (8 points) Find $g'(t)$ given that $g(t) = \cos (\tan (t^8))$

$$g'(t) = -\sin (\tan (t^8)) \cdot (\tan (t^8))'$$
$$= -\sin (\tan (t^8)) \cdot \sec^2 (t^8) \cdot (t^8)'$$
$$= -\sin (\tan (t^8)) \cdot \sec^2 (t^8) \cdot 8t^7$$

6. (12 points) A man is standing on a bridge over a river. He reaches over the railing and throws a stone vertically upward. Until it lands in the river, the stone’s height in feet above the river is $h = -16t^2 + 24t + 40$ where $t$ is measured in seconds since the stone was thrown. What is the velocity of the stone as it strikes the river? Simplify your answer.

Note that $h = 0$ when the stone strikes the river,

$$0 = -16t^2 + 24t + 40$$
$$0 = -8(2t^2 - 3t - 5)$$
$$0 = -8(t+1)(2t-5)$$
$$t = -1 \text{ or } t = \frac{5}{2}$$

not applicable

velocity is $h' = -32t + 24$

$$h'\left(\frac{5}{2}\right) = -32\left(\frac{5}{2}\right) + 24$$
$$= -80 + 24$$
$$= -56 \text{ ft/s}$$
7. (9 points) What is the slope of the line tangent to the graph of \( f(x) = \frac{\sin(2x) \cos x}{1 - \sin^2 x} \) at \( x = \pi/3 \)? Simplify your answer.

Since \( \sin(2x) = 2\sin x \cos x \) and \( 1 - \sin^2 x = \cos^2 x \), we simplify to get:

\[
f(x) = \frac{\sin(2x) \cos x}{1 - \sin^2 x} = \frac{2\sin x \cos x \cos x}{\cos^2 x} = 2\sin x
\]

Thus \( f'(x) = 2\cos x \)

and the slope at \( x = \pi/3 \) is:

\[
f'(\pi/3) = 2\cos(\pi/3) = 2 \cdot \frac{1}{2} = 1
\]

8. (9 points) Find \( \frac{dy}{dx} \) given that \( x^3 y^2 + x^5 = \sin(y^3) \). It is okay to leave your answer in terms of both \( x \) and \( y \).

\[
\frac{d}{dx} (x^3 y^2 + x^5) = \frac{d}{dx} (\sin(y^3))
\]

\[
3x^2 y^2 + x^3 2y \frac{dy}{dx} + 5x^4 = \cos(y^3) \cdot 3y^2 \frac{dy}{dx}
\]

\[
2x^3 y \frac{dy}{dx} - 3y^2 \cos(y^3) \frac{dy}{dx} = -5x^4 - 3x^2 y^2
\]

\[
\left(2x^3 y - 3y^2 \cos(y^3) \right) \frac{dy}{dx} = -5x^4 - 3x^2 y^2
\]

\[
\frac{dy}{dx} = \frac{-5x^4 - 3x^2 y^2}{2x^3 y - 3y^2 \cos(y^3)}
\]
9. (8 points) Evaluate the following limit.
\[
\lim_{{x \to 0}} \frac{e^{3x} - 3x - 1}{5x^2} = \lim_{{x \to 0}} \frac{3e^{3x} - 3}{10x} = \lim_{{x \to 0}} \frac{9e^{3x}}{10} = \frac{9}{10}
\]

Each circle refers to an application of L'Hopital's rule.

10. (12 points) For the given function, determine the intervals upon which it is increasing/decreasing, as well as the x-coordinate for each local maxima/minima.

\[
f(x) = \frac{x - 1}{x^2 + 24}
\]

\[
f'(x) = \frac{(x-1)'(x^2+24) - (x-1)(x^2+24)'}{(x^2+24)^2}
\]

\[
= \frac{-x^2 + 2x + 24}{(x^2+24)^2}
\]

\[
= -(x-6)(x+4)
\]

\[
\begin{array}{c|c|c}
\text{f decr. on } (-\infty, -4] & \text{f incr. on } [-4, 6] & \text{f decr. on } [6, \infty) \\
\text{by first deriv. test, } & & \\
F \text{ has a local min } & & \\
\text{at } x = -4 & & \text{and a local max} \\
\text{and a local max } & & \text{at } x = 6 \\
\end{array}
\]
11. (10 points) For each \( x > 0 \), a triangle is formed with vertices \((0, 0)\), \((x, 3e^{-2x})\) and \((x, -5e^{-2x})\). What is the value of \( x \) which results in the triangle of largest area?

\[
\begin{align*}
A &= \frac{1}{2}(x)(8e^{-2x}) \\
A &= 4xe^{-2x} \\
A' &= (4x)'(e^{-2x}) + (4x)(e^{-2x})' \\
A' &= 4e^{-2x} - 8xe^{-2x} \\
A' &= 4e^{-2x}(1 - 2x) \\
A' &= 0 \text{ at } x = \frac{1}{2}
\end{align*}
\]

Values of \( A' \):

\[\xrightarrow{++} \frac{1}{2} \xrightarrow{--} x \]

Max area occurs at \( x = \frac{1}{2} \) where area is

\[
4\left(\frac{1}{2}\right)e^{-2\left(\frac{1}{2}\right)} = \frac{2}{e}
\]
Students – do not write on this page!

1. (8 points) ______________________

2. (8 points) ______________________

3. (8 points) ______________________

4. (8 points) ______________________

5. (8 points) ______________________

6. (12 points) ____________________

7. (9 points) ______________________

8. (9 points) ______________________

9. (8 points) ______________________

10. (12 points) ____________________

11. (10 points) ____________________

TOTAL (100 points) ____________