Math 220 – Trigonometry Homework #1 (solutions)

Complete the following problems by Monday's lecture. These are not to be turned in.

1. Convert from degrees to radians and simplify.

   (a) $120° = \frac{2\pi}{3}$ rad

   (b) $225° = \frac{5\pi}{4}$ rad

   (c) $-30° = -\frac{\pi}{6}$ rad

   (d) $300° = \frac{5\pi}{3}$ rad

   (e) $540° = 3\pi$ rad

   (f) $2° = \frac{\pi}{90}$ rad

   multiply by $\frac{\pi \text{ rad}}{180°}$ or think about position on unit circle

2. Convert from radians to degrees and simplify.

   (a) $\pi/3$ rad $= 60°$

   (b) $\pi/12$ rad $= 15°$

   (c) $7\pi/4$ rad $= 315°$

   (d) 3.5π rad $= 630°$

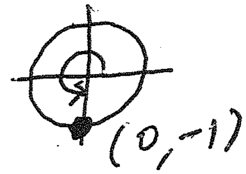
   (e) $-3\pi/2$ rad $= -270°$

   (f) 2 rad $= \frac{360}{\pi}°$

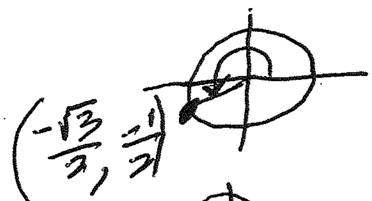
   multiply by $\frac{180°}{\pi \text{ rad}}$ or think about position on unit circle unit circles

3. Evaluate the following quantities.

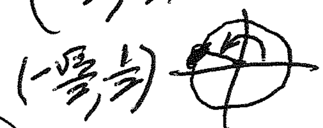
(a) $\sin(3\pi/2)$ $= -1$



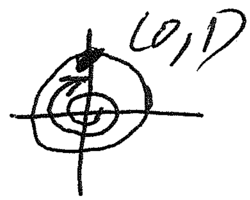
(b) $\cos(7\pi/6)$ $= -\frac{\sqrt{3}}{2}$



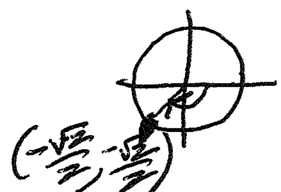
(c) $\tan(5\pi/6)$ $= \dfrac{\sin\left(\frac{5\pi}{6}\right)}{\cos\left(\frac{5\pi}{6}\right)} = \dfrac{1/2}{-\sqrt{3}/2} = \dfrac{-1}{\sqrt{3}}$



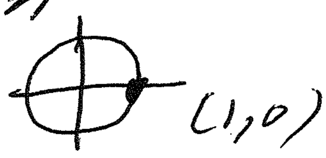
(d) $\sin(-7\pi/2)$ $= 1$



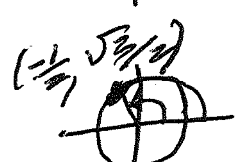
(e) $\cos(-3\pi/4)$ $= -\frac{\sqrt{2}}{2}$



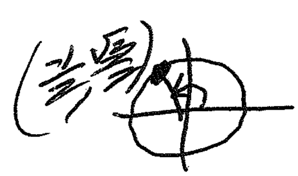
(f) $\tan(18\pi)$ $= \dfrac{\sin(18\pi)}{\cos(18\pi)} = \dfrac{0}{1} = 0$



(g) $\cot(2\pi/3)$ $= \dfrac{\cos(2\pi/3)}{\sin(2\pi/3)} = \dfrac{-1/2}{\sqrt{3}/2} = \dfrac{-1}{\sqrt{3}}$



(h) $\sec(2\pi/3)$ $= \dfrac{1}{\cos(2\pi/3)} = \dfrac{1}{-1/2} = -2$



(i) $\csc(2\pi/3)$ $= \dfrac{1}{\sin(2\pi/3)} = \dfrac{1}{\sqrt{3}/2} = \dfrac{2}{\sqrt{3}}$

4. Determine real numbers a and b so that the expression $8\sin^2\theta + 2\cos^2\theta$ can be rewritten as $a\sin^2\theta + b$.

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \cos^2\theta = 1 - \sin^2\theta$

Thus

$8\sin^2\theta + 2\cos^2\theta = 8\sin^2\theta + 2(1-\sin^2\theta)$

$= 6\sin^2\theta + 2$

so $a = 6$ and $b = 2$

5. Determine real numbers a and b so that the expression $2\tan^2\theta + 3\sec^2\theta$ can be rewritten as $a\tan^2\theta + b$.

$2\tan^2\theta + 3\sec^2\theta = 2\tan^2\theta + 3(\tan^2\theta + 1)$

$= 5\tan^2\theta + 3$

so $a = 5$ and $b = 3$

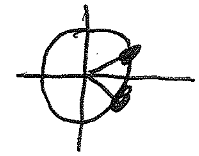
6. Simplify the expression $\cot^2\theta - \csc^2\theta$.

$\cot^2\theta - \csc^2\theta = \cot^2\theta - (\cot^2\theta + 1)$

$= -1$

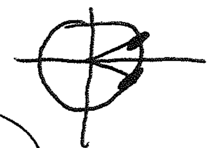
7. Determine if the following functions are even, odd or neither.

(a) $f(\theta) = \cos\theta$



$$f(-\theta) = \cos(-\theta)$$
$$= \cos\theta$$
$$= f(\theta)$$
so f is even

(b) $g(t) = \sin t$



$$g(-t) = \sin(-t)$$
$$= -\sin t$$
$$= -g(t)$$
so g is odd

(c) $h(\alpha) = \tan\alpha$

$$h(-\alpha) = \tan(-\alpha) = \frac{\sin(-\alpha)}{\cos(-\alpha)}$$
$$= \frac{-\sin(\alpha)}{\cos(\alpha)} = -\tan\alpha = -h(\alpha)$$
so h is odd

(d) $F(x) = \cot x$

$$F(-x) = \cot(-x) = \frac{\cos(-x)}{\sin(-x)} = \frac{\cos x}{-\sin x} = -\cot x = -F(x)$$
so F is odd

(e) $v(x) = \sec x$

$$v(-x) = \sec(-x) = 1/\cos(-x) = 1/\cos x = \sec x = v(x)$$
so v is even

(f) $f(x) = \csc x$

$$f(-x) = \csc(-x) = \frac{1}{\sin(-x)} = \frac{1}{-\sin x} = -\csc x = -f(x)$$
so f is odd

(g) $g(\beta) = \beta^4 \sin\beta$

$$g(-B) = (-B)^4 \sin(-B) = B^4(-\sin B) = -B^4 \sin B$$
$$= -g(B)$$
so g is odd

(h) $h(x) = \sin^3 x \tan x$

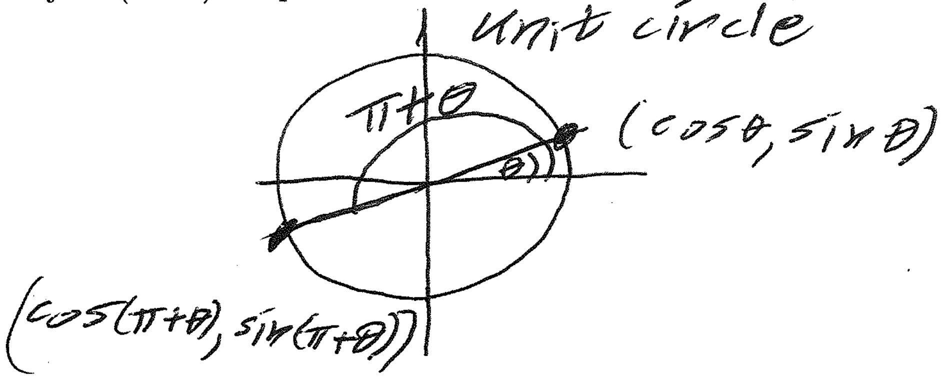
$$h(-x) = \sin^3(-x)\tan(-x) = (-\sin x)^3(-\tan x) = \sin^3 x \tan x$$
$$= h(x)$$
so h is even

(i) $w(x) = (\sin x + \cos x)^2$

$$w(x) = \sin^2 x + 2\sin x\cos x + \cos^2 x$$
$$w(x) = 1 + 2\sin x\cos x$$
$$w(-x) = 1 + 2\sin(-x)\cos(-x)$$
$$= 1 + 2(-\sin x)(\cos x)$$
$$= 1 - 2\sin x\cos x$$

$$w(-x) \neq w(x)$$
$$w(-x) \neq -w(x)$$
w is neither even nor odd

8. For all θ, the quantity $\sin(\pi+\theta)$ is equivalent to which one of the following?



(a) 0

(b) 1

(c) $\sin\theta$

(d) $-\sin\theta$

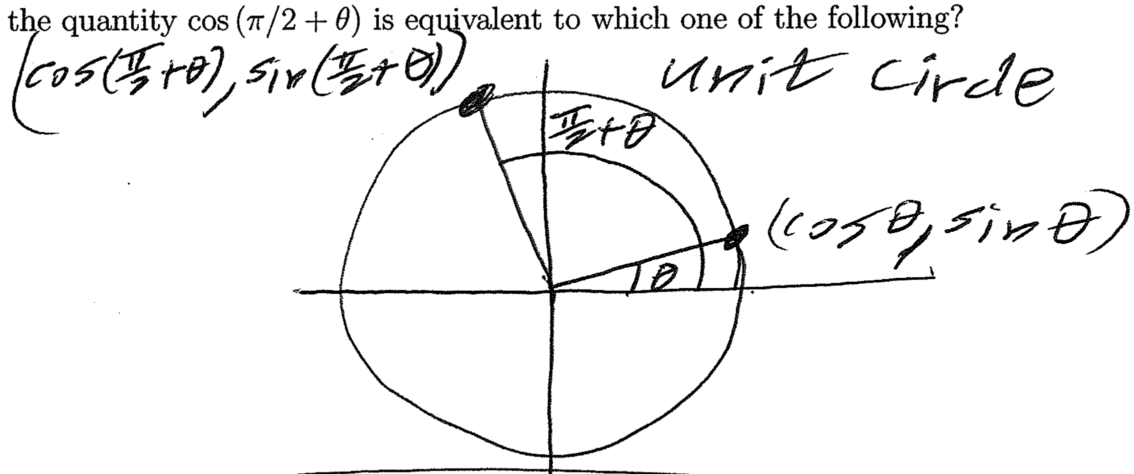
(e) $\cos\theta$

(f) $-\cos\theta$

or from identity 12a in appendix D, obtain

$$\sin(\pi+\theta) = \sin\pi\cos\theta + \cos\pi\sin\theta$$
$$= 0\cdot\cos\theta + (-1)\cdot\sin\theta$$
$$= -\sin\theta$$

9. For all θ, the quantity $\cos(\pi/2+\theta)$ is equivalent to which one of the following?



(a) 0

(b) 1

(c) $\sin\theta$

(d) $-\sin\theta$

(e) $\cos\theta$

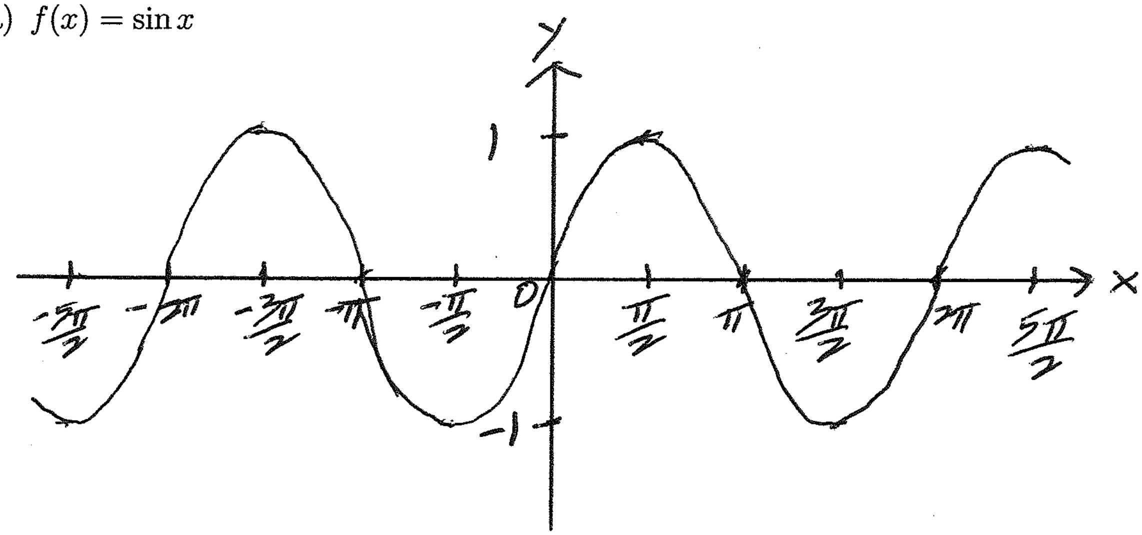
(f) $-\cos\theta$

or from identity 12b in appendix D, obtain

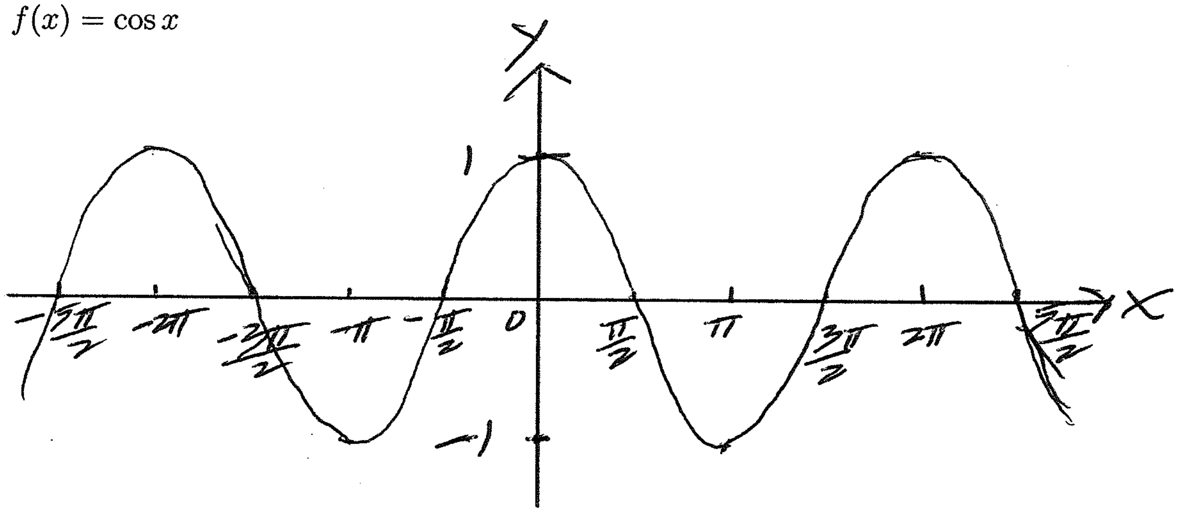
$$\cos(\tfrac{\pi}{2}+\theta) = \cos\tfrac{\pi}{2}\cos\theta - \sin\tfrac{\pi}{2}\sin\theta$$
$$= 0\cdot\cos\theta - 1\cdot\sin\theta$$
$$= -\sin\theta$$

10. Carefully sketch a graph of each of the following functions. Show more than one period for each function and include x-intercepts and y-intercepts.

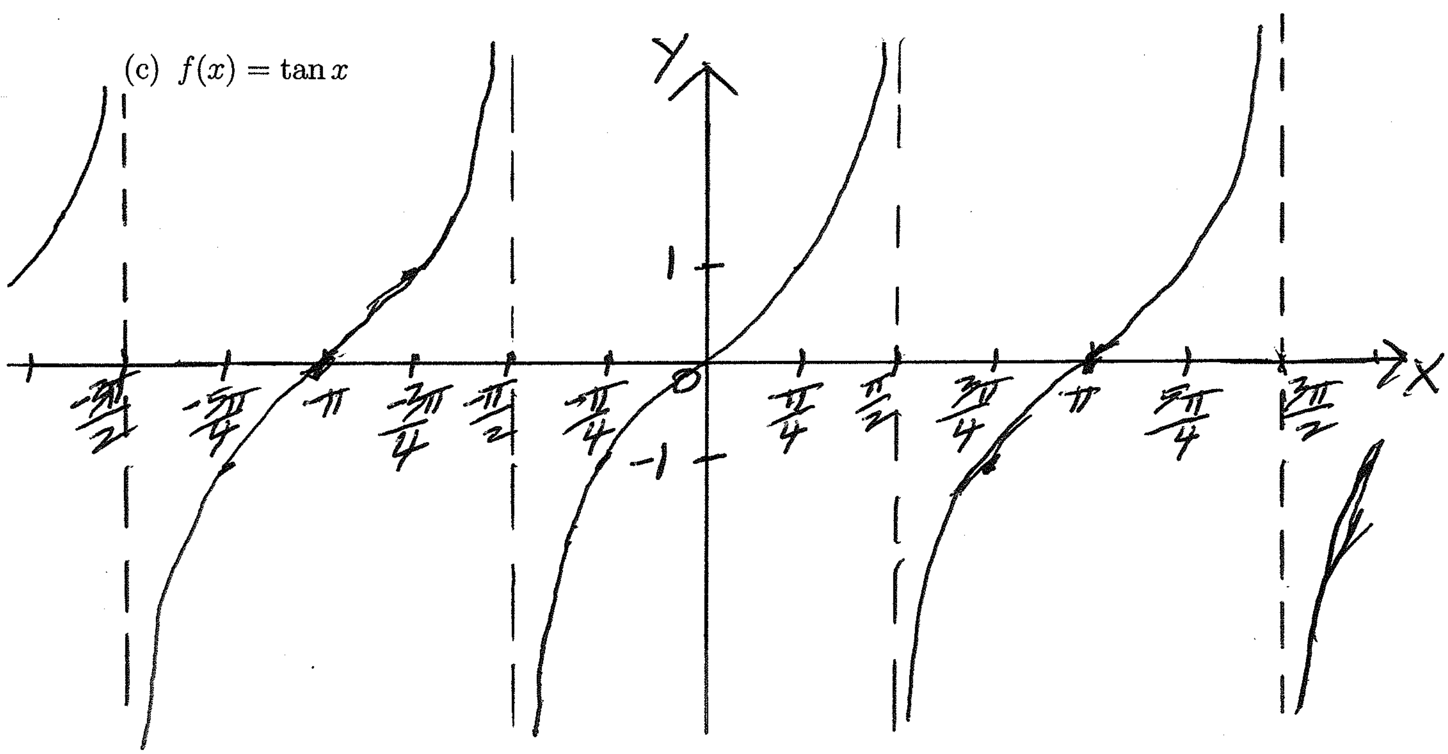
(a) $f(x) = \sin x$



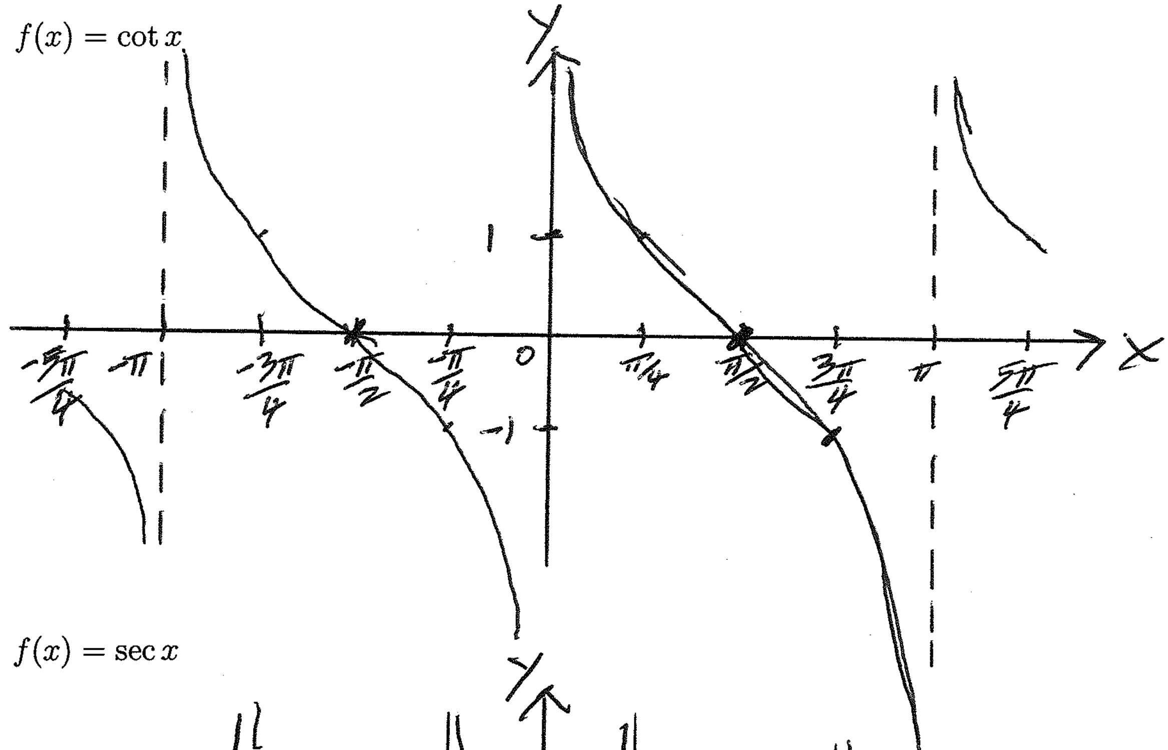
(b) $f(x) = \cos x$



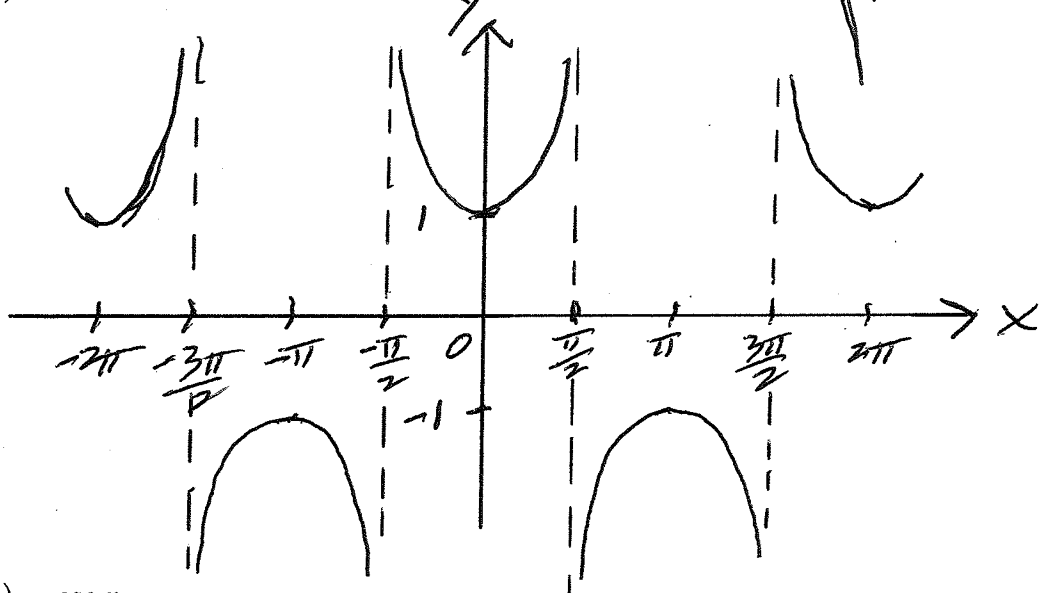
(c) $f(x) = \tan x$



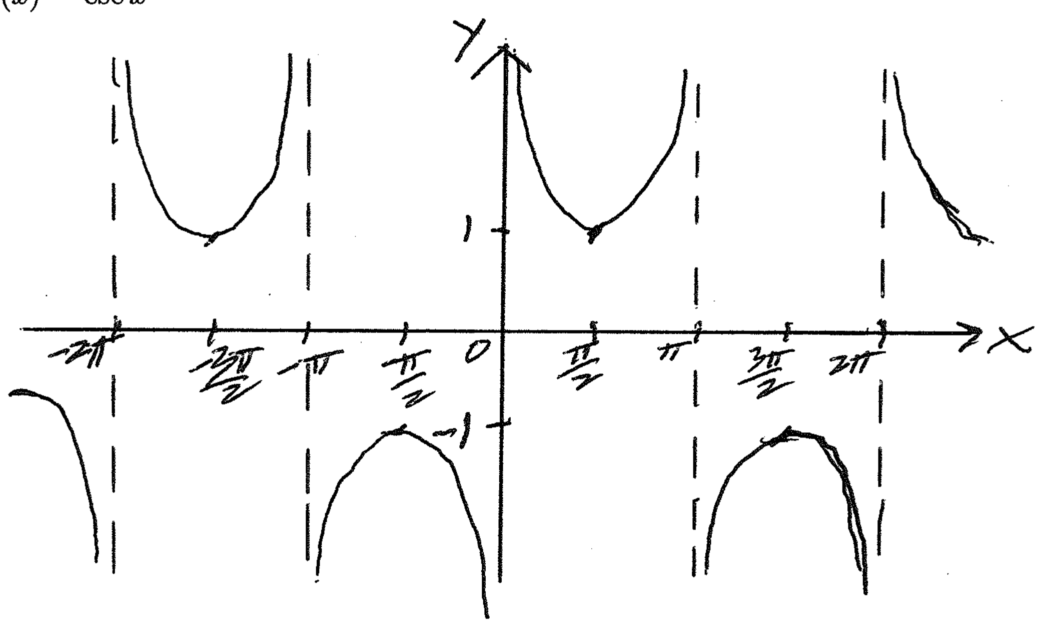
(d) $f(x) = \cot x$



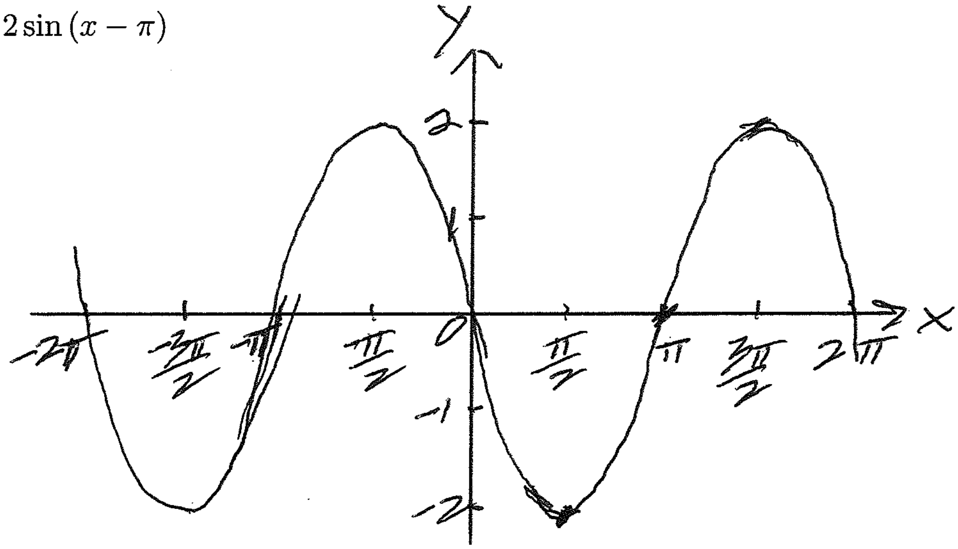
(e) $f(x) = \sec x$



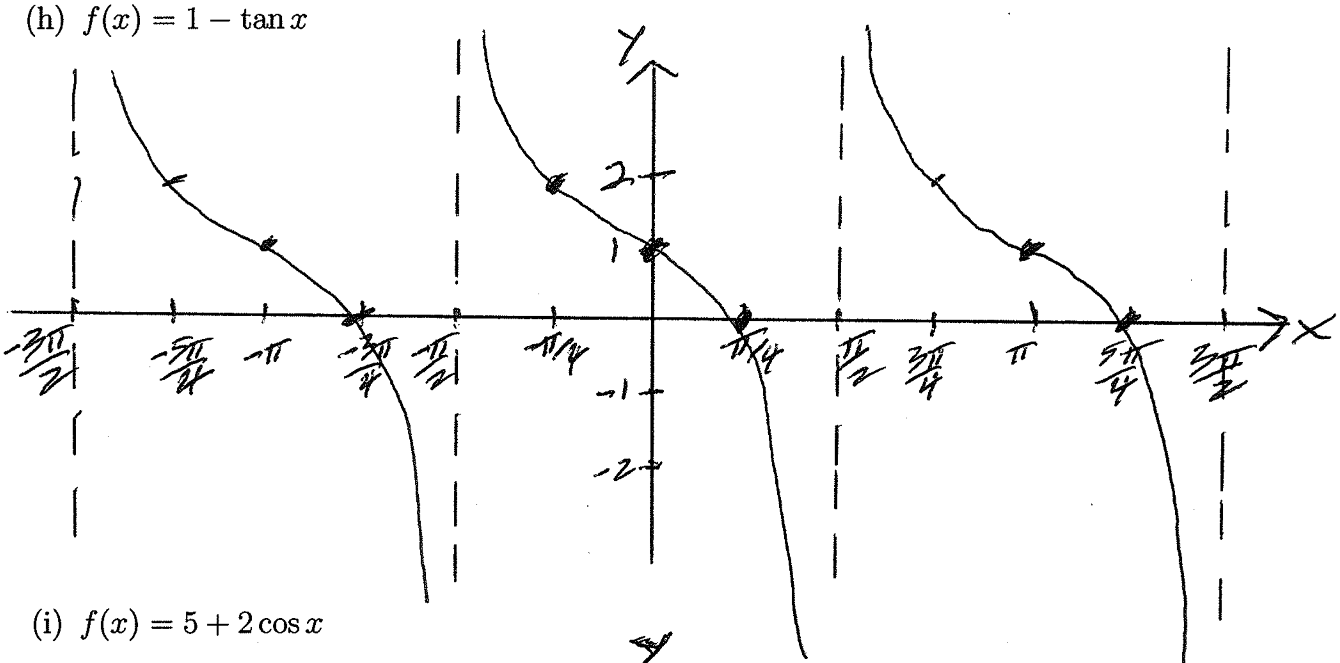
(f) $f(x) = \csc x$



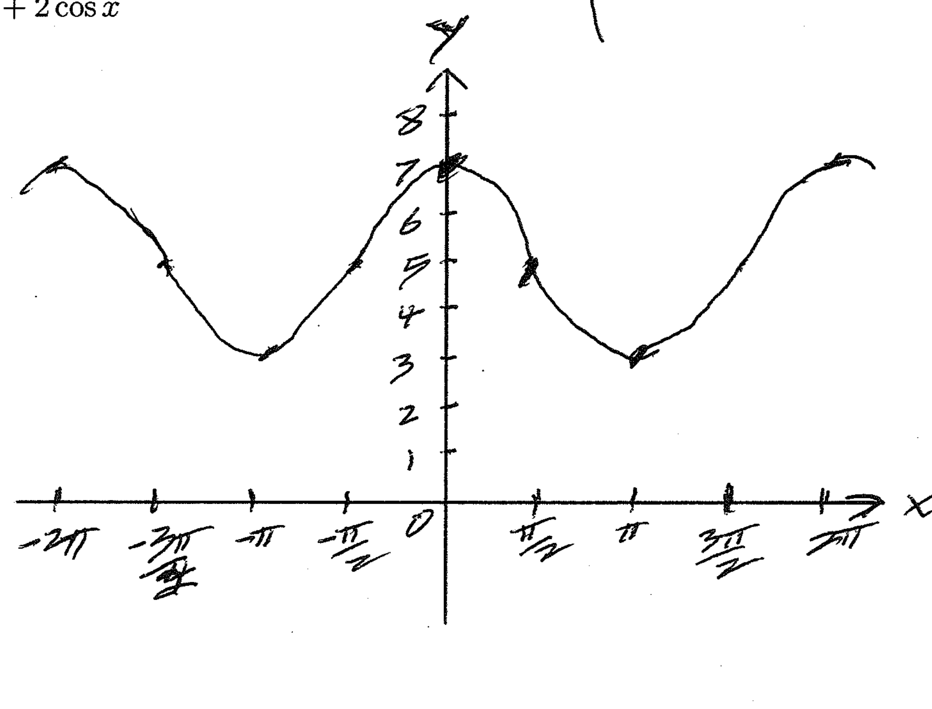
(g) $f(x) = 2\sin(x - \pi)$



(h) $f(x) = 1 - \tan x$



(i) $f(x) = 5 + 2\cos x$



Appendix D (solutions)

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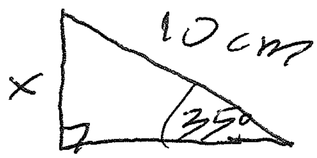

$$\sin(35^\circ) = \frac{x}{10}$$

$$x = 10\sin(35^\circ)$$

$$x \approx 5.74 \text{ cm}$$

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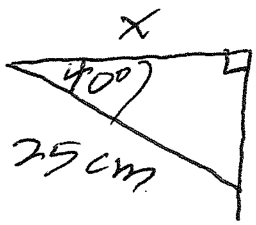

$$\cos(40^\circ) = \frac{x}{25}$$

$$x = 25\cos(40^\circ)$$

$$x \approx 19.15 \text{ cm}$$

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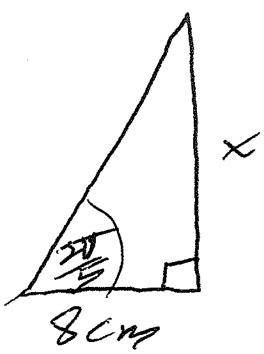

$$\tan\left(\frac{2\pi}{5}\right) = \frac{x}{8}$$

$$x = 8\tan\left(\frac{2\pi}{5}\right)$$

$$x \approx 24.62 \text{ cm}$$

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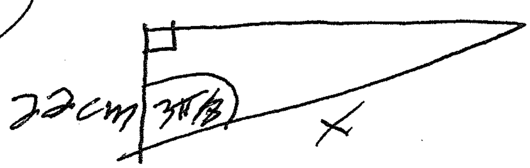

$$\cos\left(\frac{3\pi}{8}\right) = \frac{22}{x}$$

$$x\cos\left(\frac{3\pi}{8}\right) = 22$$

$$x = \frac{22}{\cos\left(\frac{3\pi}{8}\right)}$$

$$x \approx 57.49 \text{ cm}$$