

Name

SOLUTIONS

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 15 minutes for this quiz.

1. (2 points) Find the most general antiderivative of the function $f(x) = \frac{9x^5 + 5x^2 + 2}{x^3}$

$$f(x) = \frac{9x^5}{x^3} + \frac{5x^2}{x^3} + \frac{2}{x^3}$$

$$f(x) = 9x^2 + \frac{5}{x} + 2x^{-3}$$

$$F(x) = \begin{cases} 3x^3 + 5\ln|x| - x^{-2} + C_1, & x < 0 \\ 3x^3 + 5\ln|x| - x^{-2} + C_2, & x > 0 \end{cases}$$

2. (4 points) Find a formula for $f(x)$ given that $f''(x) = 5e^x - 4\sin x$, $f(0) = 15$ and $f'(0) = 12$.

$$f'(x) = 5e^x + 4\cos x + C_1$$

$$12 = f'(0) = 5e^0 + 4\cos(0) + C_1$$

$$12 = 9 + C_1 \Rightarrow C_1 = 3$$

$$f'(x) = 5e^x + 4\cos x + 3$$

$$f(x) = 5e^x + 4\sin x + 3x + C_2$$

$$15 = f(0) = 5e^0 + 4\sin(0) + 3(0) + C_2$$

$$15 = 5 + C_2 \Rightarrow C_2 = 10$$

$$f(x) = 5e^x + 4\sin x + 3x + 10$$

3. (4 points) Due to a terrible storm, the water in a river is rising. The people in a nearby town are worried that flooding will occur. They have sandbags along both sides of the river which will stop the flooding as long as the total rise in the water-level is less than 14 inches. The rate at which the water is rising is decreasing until the storm finally stops 24 hours later, and is recorded every 6 hours in the table below.

# hours	0	6	12	18	24
# inches per hour	1.5	0.5	0.3	0.2	0

Using as much of the given information as possible, determine

- (a) an upper estimate for the number of inches that the water-level of the river rises during this 24-hour period. Simplify your answer.

$$\begin{aligned}
 & (1.5 \frac{\text{in}}{\text{hr}})(6 \text{ hrs}) + (0.5 \frac{\text{in}}{\text{hr}})(6 \text{ hrs}) + (0.3 \frac{\text{in}}{\text{hr}})(6 \text{ hrs}) + (0.2 \frac{\text{in}}{\text{hr}})(6 \text{ hrs}) \\
 &= 6(1.5 + 0.5 + 0.3 + 0.2) \text{ inches} \\
 &= 6(2.5) = \boxed{15 \text{ inches}}
 \end{aligned}$$

- (b) a lower estimate for the number of inches that the water-level of the river rises during this 24-hour period. Simplify your answer.

$$\begin{aligned}
 & (0.5 \frac{\text{in}}{\text{hr}})(6 \text{ hrs}) + (0.3 \frac{\text{in}}{\text{hr}})(6 \text{ hrs}) + (0.2 \frac{\text{in}}{\text{hr}})(6 \text{ hrs}) + (0 \frac{\text{in}}{\text{hr}})(6 \text{ hrs}) \\
 &= 6(0.5 + 0.3 + 0.2 + 0) \text{ inches} \\
 &= \boxed{6 \text{ inches}}
 \end{aligned}$$

It rains between 6 and 15 inches.
 There is a possibility of flooding,
 but a good chance that the town
 does not flood.