Math 220 Quiz 7 (take-home) Spring 2011

Name SOLUTIONS

- You may work with other students in this class. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes or the textbook.
- No calculators or computers are allowed on any problem.
- You must show sufficient work to justify each answer.
- The quiz should be turned in to your TA at the beginning of your discussion section meeting on Monday, March 7 (Merit sections) or Tuesday, March 8 (other sections).
- Be sure that the pages are nicely stapled – do not just fold the corners.
- Note to TA's – you should not help students with these specific problems or go over solutions until after 4pm Tuesday.

1. (2 points) Given that $w = \left(t^2+3\right)^{\sin t}$, find a formula for $\frac{dw}{dt}$. Your formula must be written in terms of t.

$$\ln(w) = \ln\left((t^2+3)^{\sin t}\right)$$

$$\ln(w) = \sin t \ln(t^2+3)$$

$$\frac{1}{w}\frac{dw}{dt} = (\cos t)(\ln(t^2+3)) + (\sin t)\left(\frac{1}{t^2+3}\cdot 2t\right)$$

$$\frac{dw}{dt} = w\left((\cos t)(\ln(t^2+3)) + \frac{2t\sin t}{t^2+3}\right)$$

$$\frac{dw}{dt} = (t^2+3)^{\sin t}\left(\cos t\ln(t^2+3) + \frac{2t\sin t}{t^2+3}\right)$$

2. (3 points) Given that $y = \tan^{-1}(xy)$, find a formula for $\dfrac{dy}{dx}$. Your formula may be written in terms of x, y, or both variables.

METHOD 1 $y = \tan^{-1}(xy)$

$$\tan y = xy$$

$$x = \tan y / y$$

$$\frac{dx}{dy} = \frac{(\sec^2 y)(y) - (\tan y)(1)}{y^2}$$

$$\frac{dy}{dx} = \frac{1}{dx/dy} = \frac{y^2}{y\sec^2 y - \tan y}$$

METHOD 2

$$y = \tan^{-1}(xy)$$

$$\frac{dy}{dx} = \frac{1}{1+(xy)^2} \cdot \frac{d}{dx}(xy)$$

$$\frac{dy}{dx} = \frac{1}{1+x^2y^2} \cdot \left((1)(y) + (x)\left(\frac{dy}{dx}\right)\right)$$

$$\frac{dy}{dx}(1+x^2y^2) = y + x\frac{dy}{dx}$$

$$\frac{dy}{dx}(1+x^2y^2 - x) = y$$

$$\frac{dy}{dx} = \frac{y}{1+x^2y^2-x}$$

3. (2 points) A rock is thrown upward from the surface of the moon. Between the time that the rock is thrown and the time that the rock hits the ground, the rock's height is given by the formula $s(t) = 24t - 0.8t^2$, where t is the number of seconds since the rock is first thrown and $s(t)$ is measured in meters above the moon's surface.

(a) Find a formula for the rock's velocity at time t.

$v(t) = s'(t) = 24 - 1.6t$

(b) Find a formula for the rock's acceleration at time t.

$a(t) = s''(t) = -1.6$

(c) How fast was the rock moving, and in which direction, at the end of 25 seconds?

$v(25) = s'(25) = 24 - 1.6(25)$
$= -16$ m/sec

it was moving toward the surface at 16 m/sec

(d) What is the rock's maximum height and at what time does it reach that height?

we set velocity $= 0$.

$0 = 24 - 1.6t$

$1.6t = 24$

$t = 24/1.6 = \frac{240}{16} = 15$

$t = 15$ sec

$s(15) = 24(15) - 0.8(15)^2$
$= 360 - 0.8(225)$
$= 360 - 180$
$= 180$ meters

4. (3 points) The graph of a function $y = f(x)$ has the property that the slope of the tangent line at each point on this graph is equal to one half its y-coordinate. If the graph goes through the point $(6, e^5)$, then find a formula for $f(x)$.

$$\frac{dy}{dx} = \frac{1}{2}y \text{ and } y(6) = e^5$$

$$y = Ce^{\frac{1}{2}x}$$

$$e^5 = Ce^{\frac{1}{2}(6)}$$

$$e^5 = Ce^3$$

$$C = e^5/e^3 = e^2$$

$$y = e^2 e^{\frac{1}{2}x}$$

$$y = e^{2+\frac{1}{2}x}$$