

Name SOLUTIONS

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 15 minutes for this quiz.

1. (3 points each) Differentiate the following functions.

(a)  $h = \frac{\cot x}{x^5}$

$$\frac{dh}{dx} = \frac{(\cot x)' x^5 - \cot x (x^5)'}{(x^5)^2}$$

$$\frac{dh}{dx} = \frac{(-\csc^2 x) x^5 - \cot x (5x^4)}{x^{10}}$$

$$\frac{dh}{dx} = \frac{-x \csc^2 x - 5 \cot x}{x^6}$$

(b)  $v(x) = x^3 e^x \cos x$

$$v'(x) = (x^3)' e^x \cos x + x^3 (e^x)' \cos x + x^3 e^x (\cos x)'$$

$$v'(x) = 3x^2 e^x \cos x + x^3 e^x \cos x - x^3 e^x \sin x$$

2. (4 points) Find the equation for any line which is tangent to the graph of

$$y = 10 + 6x - 3 \tan x$$

and perpendicular to the line  $x + 2y = 8$ .

$$x + 2y = 8$$

$$2y = -x + 8$$

$$y = -\frac{1}{2}x + 4 \text{ has slope } -\frac{1}{2}$$

A line perpendicular has slope  $\frac{-1}{-\frac{1}{2}} = 2$

we need a line tangent to

$$y = 10 + 6x - 3 \tan x \text{ with slope } 2$$

$$y' = 6 - 3 \sec^2 x$$

$$2 = 6 - 3 \sec^2 x$$

$$3 \sec^2 x = 4$$

$$\sec^2 x = \frac{4}{3}$$

$$\cos^2 x = \frac{3}{4}$$

$$\cos x = \pm \frac{\sqrt{3}}{2}$$

one such value for  $x$  is  $\frac{\pi}{6}$

$$y\left(\frac{\pi}{6}\right) = 10 + 6\left(\frac{\pi}{6}\right) - 3 \tan\left(\frac{\pi}{6}\right)$$

$$= 10 + \pi - 3\left(\frac{1/\sqrt{3}}{1/2}\right) = 10 + \pi - \sqrt{3}$$

POINT:  $\left(\frac{\pi}{6}, 10 + \pi - \sqrt{3}\right)$  SLOPE: 2

$$\text{EQUATION: } y - (10 + \pi - \sqrt{3}) = 2\left(x - \frac{\pi}{6}\right)$$

$$y = 2x + 10 + \frac{2\pi}{3} - \sqrt{3}$$