Math 220 (section ???) Quiz 3 Spring 2011

Name ______________________________

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 15 minutes for this quiz.

1. (2 points) Show precisely how the *Intermediate Value Theorem* is used to prove that the equation $x^3 - 2x - 3 = 0$ has at least one real solution.

Let $f(x) = x^3 - 2x - 3$

$f(1) = -4$, $f(2) = 1$

f is a polynomial so is continuous everywhere. In particular, f is continuous on $[1, 2]$. Since $f(1) < 0$ and $f(2) > 0$, by the Intermediate Value Theorem there is a number c in $(1, 2)$ such that $f(c) = 0$.

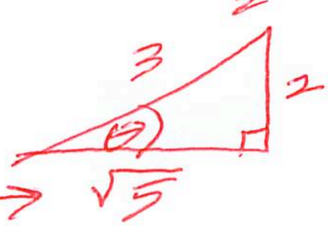
Thus $x^3 - 2x - 3 = 0$ has c as a solution with $1 < c < 2$.

2. (2 points) Evaluate $\cot\left(\sin^{-1}\left(\frac{2}{3}\right)\right)$.

Let $\theta = \sin^{-1}\left(\frac{2}{3}\right)$

so $\sin\theta = \frac{2}{3}$ $\left(\frac{\text{opp}}{\text{hyp}}\right)$

Noting that $0 < \theta < \frac{\pi}{2}$ we have



Pythagorean Theorem → $\sqrt{5}$

$\cot\left(\sin^{-1}\left(\frac{2}{3}\right)\right) = \cot\theta$

$= \frac{\sqrt{5}}{2}$ $\left(\frac{\text{adj}}{\text{opp}}\right)$

3. (2 points each) Evaluate the following limits.

(a) $\lim_{x \to 0.5^+} \frac{\ln x}{2x - 1}$ $\ln x \to \ln(0.5)$ which is negative; $2x-1 \to 0+$

Thus $\lim_{x \to 0+} \frac{\ln x}{2x-1} = -\infty$

(b) $\lim_{x \to 2} \frac{x^2 + 3x - 10}{x^2 - 4}$ $x^2+3x-10 \to 0$; $x^2-4 \to 0$

$$\lim_{x \to 2} \frac{x^2+3x-10}{x^2-4} = \lim_{x \to 2} \frac{(x-2)(x+5)}{(x-2)(x+2)}$$

$$= \lim_{x \to 2} \frac{x+5}{x+2}$$

$$= \frac{7}{4}$$

(c) $\lim_{x \to \infty} \frac{6 + 8e^x}{2e^x}$ $6+8e^x \to \infty$; $2e^x \to \infty$

$$\lim_{x \to \infty} \frac{6+8e^x}{2e^x} = \lim_{x \to \infty} \left(\frac{6}{2e^x} + \frac{8e^x}{2e^x}\right)$$

$$= \lim_{x \to \infty} \left(\frac{3}{e^x} + 4\right)$$

$$= 4$$