

Name _____

Solutions

• You have 15 minutes

• No calculators

• Show sufficient work

1. (4 points) Find the average value of the function $f(x) = \tan x \sec^2 x$ on the interval $[\pi/6, \pi/4]$. Simplify your answer.

$$f_{\text{ave}} = \frac{1}{\frac{\pi}{4} - \frac{\pi}{6}} \int_{\pi/6}^{\pi/4} \tan x \sec^2 x \, dx$$

$$= \frac{24}{6\pi - 4\pi} \int_{1/\sqrt{3}}^1 u \, du$$

$$= \frac{12}{\pi} \cdot \left[\frac{1}{2} u^2 \right]_{1/\sqrt{3}}^1$$

$$= \frac{12}{\pi} \cdot \left[\frac{1}{2} - \frac{1}{2} \cdot \frac{1}{3} \right]$$

$$= \frac{12}{\pi} \cdot \left[\frac{2}{6} \right]$$

$$= \frac{4}{\pi}$$

using substitution

$$u = \tan x$$

$$du = \sec^2 x \, dx$$

$$x = \pi/6 \Rightarrow u = \tan(\pi/6)$$

$$= \frac{1/\sqrt{3}}{1/2}$$

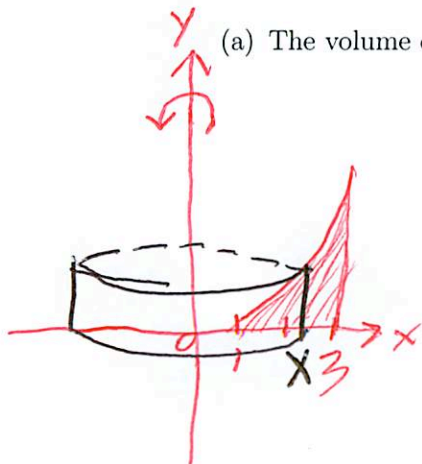
$$= \frac{1}{\sqrt{3}}$$

$$x = \pi/4 \Rightarrow u = \tan(\pi/4)$$

$$= 1$$

Let R be the region bounded by the x -axis and the graph of $y = 5x + 3e^{2x}$ on the interval $[1, 3]$. Set up, but do not evaluate, definite integrals which represent the given quantities. Use proper notation. Each problem is worth 3 points.

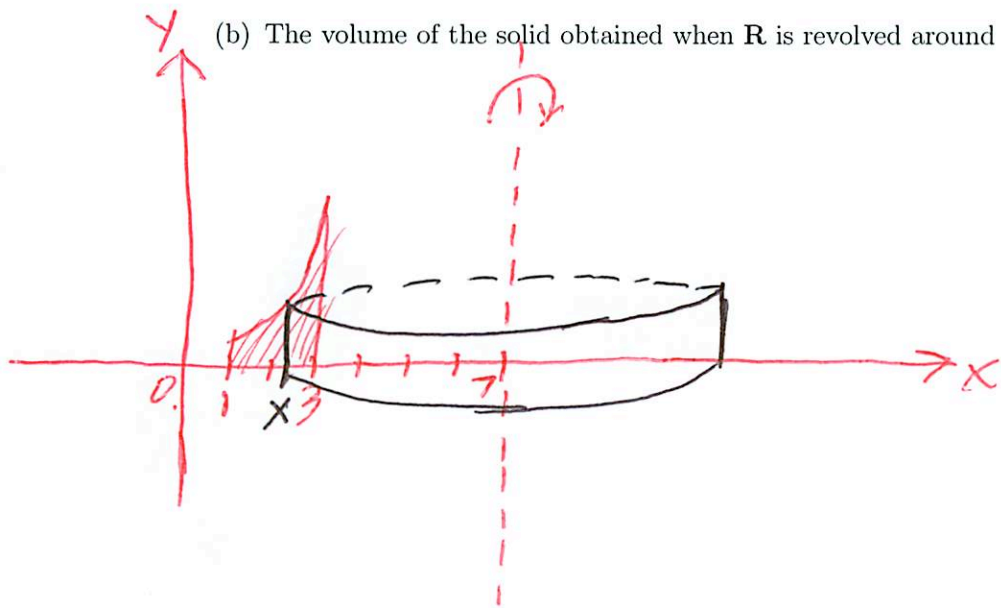
(a) The volume of the solid obtained when R is revolved around the y -axis.



$$V = \int_1^3 2\pi(\text{radius})(\text{height}) dx$$

$$V = \int_1^3 2\pi x (5x + 3e^{2x}) dx$$

(b) The volume of the solid obtained when R is revolved around the vertical line $x = 7$.



$$V = \int_1^3 2\pi(\text{radius})(\text{height}) dx$$

$$V = \int_1^3 2\pi(7-x)(5x + 3e^{2x}) dx$$