Math 220 (section ???) **Quiz 12** **Spring 2011**

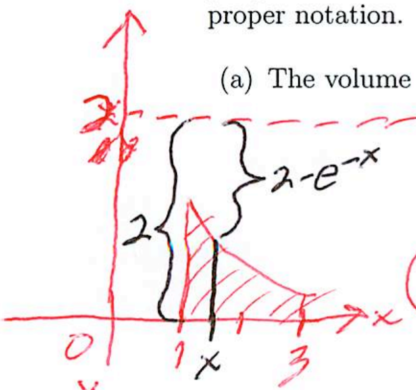
Name SOLUTIONS

- You have 15 minutes
- No calculators
- Show sufficient work

1. (3 points each) Let **R** be the region bounded by the x-axis and the graph of $y = e^{-x}$ on the interval $[1, 3]$. Set up, but do not evaluate, definite integrals which represent the given quantities. Use proper notation.

$y = e^{-x} \Rightarrow x = -\ln y$

(a) The volume of the solid obtained when **R** is revolved around the line $y = 2$.

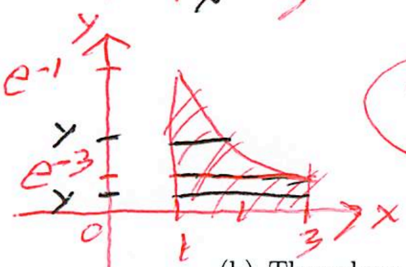


$$V = \int_1^3 \left(\pi \left(\text{big radius}\right)^2 - \pi \left(\text{little radius}\right)^2\right) dx$$

$$V = \int_1^3 \left(\pi (2)^2 - \pi (2 - e^{-x})^2\right) dx$$

OR

$$V = \int_0^{e^{-3}} 2\pi \underbrace{(2-y)}_{\text{rad.}} \underbrace{(2)}_{\text{height}} dy + \int_{e^{-3}}^{e^{-1}} 2\pi \underbrace{(2-y)}_{\text{rad}} \underbrace{(-\ln y - 1)}_{\text{height}} dy$$



(b) The volume of the solid with base **R** for which the cross-sections perpendicular to the x-axis are squares.

$$V = \int_1^3 \left(\text{side length}\right)^2 dx$$

$$V = \int_1^3 (e^{-x})^2 dx$$

2. (2 points) Precisely state the *Mean Value Theorem.*

If f is continuous on the closed interval $[a,b]$ and differential on the open interval (a,b), then there is a c in (a,b) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}.$$

3. (2 points) Explain carefully why $f(x) = x^7 + 5x^3 + 2x - 20$ cannot have two real roots.

Suppose there are two real roots r_1 and r_2. without loss of generality we can assume $r_1 < r_2$.

Being roots means $f(r_1) = f(r_2) = 0$.

Since polynomials are continuous and differentiable everywhere, we have

- f is continuous on $[r_1, r_2]$
- f is differentiable on (r_1, r_2)
- Also $f(r_1) = f(r_2)$

By Rolle's Theorem, there is a c in (r_1, r_2) such that $f'(c) = 0$. However $f'(x) = 7x^6 + 15x^2 + 2 \geq 2$ for all x so cannot equal 0, This contradicts our assumption of two real roots, Thus f does not have two real roots.