Math 220 (section ???) Quiz 11 Spring 2011

Name SOLUTIONS

- You have 15 minutes
- No calculators
- Show sufficient work

1. (2 points) Evaluate the integral $\int \frac{x^3}{\sqrt[3]{x^4+2}}\,dx$. $= \int x^3(x^4+2)^{-1/3}dx$

$u = x^4+2$

$\frac{du}{dx} = 4x^3$

$du = 4x^3dx$

$\frac{1}{4}du = x^3dx$

$= \int \frac{1}{4}u^{-1/3}du$

$= \frac{1}{4}\frac{u^{2/3}}{2/3} + C$

$= \frac{3}{8}u^{2/3} + C$

$= \frac{3}{8}(x^4+2)^{2/3} + C$

2. (3 points) Evaluate the integral $\int_{\pi/12}^{\pi/8} 8\sin^3(2x)\cos(2x)\,dx$ and simplify your answer.

$u = \sin(2x)$

$du = 2\cos(2x)dx$

$4du = 8\cos(2x)dx$

$x = \frac{\pi}{12} \Rightarrow u = \sin(\frac{\pi}{6}) = 1/2$

$x = \frac{\pi}{8} \Rightarrow u = \sin(\frac{\pi}{4}) = \sqrt{2}/2$

$= \int_{1/2}^{\sqrt{2}/2} 4u^3du$

$= \left[u^4\right]_{1/2}^{\sqrt{2}/2}$

$= \left(\frac{\sqrt{2}}{2}\right)^4 - \left(\frac{1}{2}\right)^4$

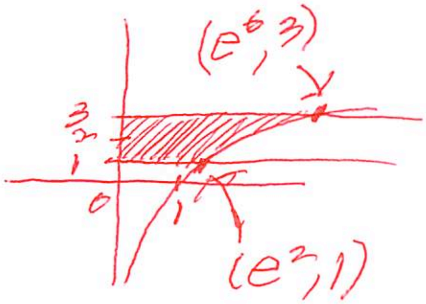
$= \frac{1}{4} - \frac{1}{16}$

$= \frac{3}{16}$

$x = e^{2y}$

3. (4 points) Consider the region from $y = 1$ to $y = 3$ between the y-axis and the graph of $y = \frac{1}{2}\ln x$. In the following manner set up, but do not evaluate, integrals which represent the area of this region.

   (a) Integrate with respect to x.



$$A = \int_0^{e^2} (3-1)\,dx + \int_{e^2}^{e^6} \left(3 - \tfrac{1}{2}\ln x\right) dx$$

$\frac{1}{2}\ln x = 3 \Rightarrow x = e^6$

$\frac{1}{2}\ln x = 1 \Rightarrow x = e^{2y}$

   (b) Integrate with respect to y. (the integrands in parts (a) and (b) should be different)

$$A = \int_1^3 e^{2y}\,dy$$

4. (1 point) Determine the area of the region in problem (3) by completing any necessary integration.

from 3b, $A = \int_1^3 e^{2y}\,dy$

$$= \left[\tfrac{1}{2}e^{2y}\right]_1^3$$

$$= \tfrac{1}{2}e^6 - \tfrac{1}{2}e^2$$