Math 220 (section ???) Quiz 10 Spring 2011

Name SOLUTIONS

- You have 15 minutes
- No calculators
- Show sufficient work

1. (3 points) Evaluate the following indefinite integral.

$$\int (1+\sqrt[3]{x})^2\,dx = \int (1+x^{1/3})^2\,dx$$

$$= \int (1+2x^{1/3}+x^{2/3})\,dx$$

$$= x + \frac{2x^{4/3}}{4/3} + \frac{x^{5/3}}{5/3} + C$$

$$= x + \frac{3}{2}x^{4/3} + \frac{3}{5}x^{5/3} + C$$

2. (3 points) Evaluate and simplify the following definite integral.

$$\int_4^{32} \frac{1}{3x}\,dx = \frac{1}{3}\int_4^{32} \frac{1}{x}\,dx$$

$$= \frac{1}{3}\Big[\ln|x|\Big]_4^{32}$$

$$= \frac{1}{3}\left[\ln(32) - \ln(4)\right]$$

$$= \frac{1}{3}\ln\left(\frac{32}{4}\right)$$

$$= \frac{1}{3}\ln(8)$$

$$= \frac{1}{3}\ln(2^3) = \frac{1}{3}\cdot 3\ln(2)$$

$$= \ln(2)$$

3. (2 points) Fill in the missing information to show that the given definite integral can be expressed as the limit of a Riemann sum. The only variables appearing in your limit should be n and k. You do not need to evaluate this limit.

$$\int_2^5 \sin\left(x^2\right) dx = \lim_{n\to\infty} \sum_{k=1}^{n} \left[\sin\left(\left(2+k\cdot\frac{3}{n}\right)^2\right)\cdot\frac{3}{n} \right]$$

$$\Delta x = \frac{5-2}{n} = \frac{3}{n}$$

$$x_k = 2 + k\cdot\Delta x$$
$$= 2 + k\cdot\frac{3}{n}$$

4. (2 points) Evaluate the following limit. Be sure to use proper notation throughout your evaluation of this limit.

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{50}{n^3} + \frac{10k}{n^2} + \frac{3}{n}\right) = \lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{50}{n^3} + \sum_{k=1}^{n}\frac{10k}{n^2} + \sum_{k=1}^{n}\frac{3}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{50}{n^3}\sum_{k=1}^{n} 1 + \frac{10}{n^2}\sum_{k=1}^{n} k + \frac{3}{n}\sum_{k=1}^{n} 1\right)$$

$$= \lim_{n\to\infty}\left(\frac{50}{n^3}\cdot n + \frac{10}{n^2}\cdot\frac{n(n+1)}{2} + \frac{3}{n}\cdot n\right)$$

$$= \lim_{n\to\infty}\left(\frac{50}{n^2} + \frac{5n+5}{n} + 3\right)$$

$$= \lim_{n\to\infty}\left(\frac{50}{n^2} + 5 + \frac{5}{n} + 3\right)$$

$$= 8$$