MATH 220 **Test 3** **Spring 2011**

Name SOLUTIONS

(circle your TA discussion section)

- ▷ **AD1**, TR 1:00-1:50, Sarah Son
- ▷ **AD4**, TR 1:00-1:50, Sogol Jahanbekam
- ▷ **AD7**, TR 3:00-3:50, Nersés Aramyan
- ▷ **AD9**, MW 9:00-10:50, Ben Reiniger
- ▷ **AD2**, TR 1:00-1:50, Daniel Hockensmith
- ▷ **AD5**, TR 2:00-2:50, Daniel Hockensmith
- ▷ **AD8**, MW 11:00-12:50, Austin Rochford

- Sit in your assigned seat (shown below).
- Do not open this test booklet until I say *START*.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- You must show sufficient work to justify each answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Quit working and close this test booklet when I say *STOP*.
- Quickly turn in your test to me or a TA and show your Student ID.

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FRONT OF ROOM – 314 Altgeld Hall

1. (8 points) Determine an appropriate linear approximation of the function $f(x) = \sqrt{x}$ and use it to approximate $\sqrt{26.3}$. Write your answer in decimal form.

$f(x) = x^{1/2}$ $\qquad f(25) = 5 \qquad$ POINT: $(25, 5)$

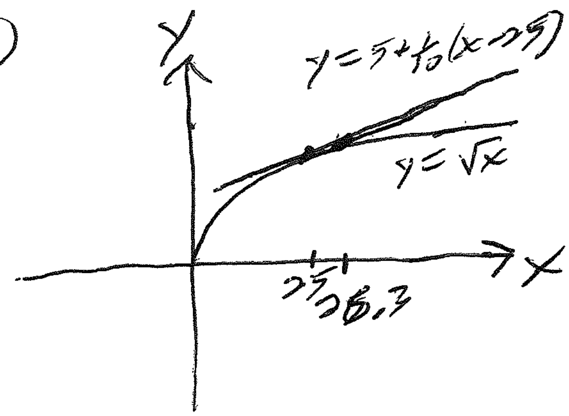
$f'(x) = \frac{1}{2}x^{-1/2}$ $\qquad f'(25) = \frac{1}{10} \qquad$ SLOPE: $1/10$

$$y - 5 = \tfrac{1}{10}(x - 25) \Rightarrow y = 5 + \tfrac{1}{10}(x - 25)$$

$\sqrt{x} \approx 5 + \frac{1}{10}(x - 25)$ for x near 25

$\sqrt{26.3} \approx 5 + \frac{1}{10}(26.3 - 25)$

$\sqrt{26.3} \approx 5.13$



2. (6 points) Precisely state *Rolle's Theorem.*

If f is continuous on $[a, b]$
and differentiable on (a, b)
and $f(a) = f(b)$
Then there is a c in (a, b)
such that $f'(c) = 0$

3. (8 points) Evaluate the following limit. Be sure to use proper notation throughout your evaluation of this limit. Simplify your answer.

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{17}{4n}-\frac{5k}{2n^2}\right) = \lim_{n\to\infty}\left(\sum_{K=1}^{n}\frac{17}{4n}-\sum_{K=1}^{n}\frac{5K}{2n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{17}{4n}\sum_{K=1}^{n}1-\frac{5}{2n^2}\sum_{K=1}^{n}K\right)$$

$$= \lim_{n\to\infty}\left(\frac{17}{4n}\cdot n-\frac{5}{2n^2}\cdot\frac{n(n+1)}{2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{17}{4}-\frac{5n+5}{4n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{17}{4}-\frac{5+\frac{5}{n}}{4}\right)$$

$$= \frac{17}{4}-\frac{5}{4} = \frac{12}{4} = 3$$

4. (12 points) Suppose f is an even function, g is an odd function, and f and g are each integrable on the interval $[-3,3]$. Given that $\int_0^3 f(x)\,dx = 5$ and $\int_0^3 g(x)\,dx = 4$, evaluate the following definite integrals.

(a) $\int_3^0 g(x)\,dx = -\int_0^3 g(x)\,dx = -4$

(b) $\int_3^3 f(x)\,dx = 0$

(c) $\int_{-3}^3 (2f(x)+4g(x))\,dx = 2\int_{-3}^3 f(x)\,dx + 4\int_{-3}^3 g(x)\,dx$

$= 2\cdot 2\int_0^3 f(x)\,dx + 0 = 4\cdot 5 = 20$

(d) $\int_{-3}^3 \left(4+(g(x))^5\right)\,dx$ $= \int_{-3}^3 4\,dx + \int_{-3}^3 (g(x))^5\,dx$

$= 24 + 0 = 24$

NOTE
$(g(-x))^5 = (-g(x))^5$
$= -(g(x))^5$
so is odd

5. (9 points each) Evaluate the following definite integrals. Simplify each answer.

(a) $\int_{\pi/3}^{\pi/2} (12 + 6\sin x)\, dx$ $= \left[12x - 6\cos x\right]_{\pi/3}^{\pi/2}$

$= \left[12\left(\frac{\pi}{2}\right) - 6\cos\left(\frac{\pi}{2}\right)\right] - \left[12\left(\frac{\pi}{3}\right) - 6\cos\left(\frac{\pi}{3}\right)\right]$

$= \left[6\pi - 0\right] - \left[4\pi - 6\left(\frac{1}{2}\right)\right]$

$= 2\pi + 3$

(b) $\int_{0}^{2} \left(6x^2 + 3e^{-x}\right)\, dx$ $= \left[2x^3 - 3e^{-x}\right]_0^2$

$= \left[2(2)^3 - 3e^{-2}\right] - \left[0 - 3\right]$

$= 19 - 3e^{-2}$

6. (8 points each) Evaluate the following indefinite integrals.

(a) $\int \frac{6x^3 + 4x^2 + 5x}{x^2}\, dx$ $= \int \left(\frac{6x^3}{x^2} + \frac{4x^2}{x^2} + \frac{5x}{x^2} \right) dx$

$= \int \left(6x + 4 + \frac{5}{x} \right) dx$

$= 3x^2 + 4x + 5\ln|x| + C$

(b) $\int \frac{1}{x\sqrt{\ln x}}\, dx$ $= \int \frac{du}{\sqrt{u}} = \int u^{-1/2}\, du$

$= 2u^{1/2} + C$

$= 2\sqrt{\ln x} + C$

$\boxed{\begin{array}{l} u = \ln x \\ du = \frac{1}{x}\, dx \end{array}}$

(c) $\int \tan^5 x \sec^4 x\, dx$

METHOD 1 $\int \tan^5 x \sec^4 x\, dx$

$= \int \tan^5 x \sec^2 x \sec^2 x\, dx$

$= \int \tan^5 x (\tan^2 x + 1) \sec^2 x\, dx$

$= \int u^5 (u^2+1)\, du$

$= \int (u^7 + u^5)\, du$

$= \frac{1}{8}u^8 + \frac{1}{6}u^6 + C$

$= \frac{1}{8}\tan^8 x + \frac{1}{6}\tan^6 x + C$

METHOD 2

$\int \tan^5 x \sec^4 x\, dx = \int \tan^4 x \sec^3 x \sec x \tan x\, dx$

$= \int (\sec^2 x - 1)^2 \sec^3 x \sec x \tan x\, dx$

$= \int (u^2-1)^2 u^3\, du$

$= \int (u^7 - 2u^5 + u^3)\, du$

$= \frac{1}{8}u^8 - \frac{1}{3}u^6 + \frac{1}{4}u^4 + C$

$= \frac{1}{8}\sec^8 x - \frac{1}{3}\sec^6 x + \frac{1}{4}\sec^4 x + C$

METHOD 3

$\int \tan^5 x \sec^4 x\, dx = \int \frac{\sin^5 x}{\cos^9 x}\, dx$

$= \int \frac{\sin^4 x \sin x}{\cos^9 x}\, dx = \int \frac{(1-\cos^2 x)^2 \sin x}{\cos^9 x}\, dx$

$= \int \frac{(1-u^2)^2}{u^9}(-du) = \int (-u^{-9} + 2u^{-7} - u^{-5})\, du$

$= \frac{1}{8}u^{-8} - \frac{1}{3}u^{-6} + \frac{1}{4}u^{-4} + C$

$= \frac{1}{8}(\cos x)^{-8} - \frac{1}{3}(\cos x)^{-6} + \frac{1}{4}(\cos x)^{-4} + C$

7. (6 points) Evaluate the following indefinite integral.

$\int x^2\sqrt{x+1}\, dx$

$u = x+1$
$du = dx$

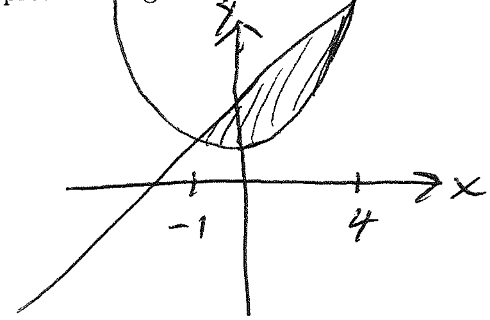
$\int x^2 \sqrt{x+1}\, dx = \int (u-1)^2 u^{1/2}\, du = \int (u^{5/2} - 2u^{3/2} + u^{1/2})\, du$

$= \frac{2}{7}u^{7/2} - \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C$

$= \frac{2}{7}(x+1)^{7/2} - \frac{4}{5}(x+1)^{5/2} + \frac{2}{3}(x+1)^{3/2} + C$

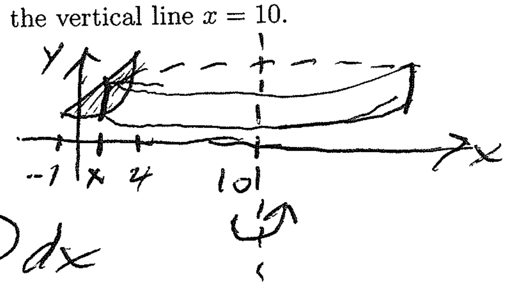
8. (6 points each) The intersection points on the graphs of $f(x) = x^2 + 2$ and $g(x) = 3x + 6$ occur at $x = -1$ and at $x = 4$. Let **R** be the finite region bounded by the graphs of $f(x)$ and $g(x)$. Set up, but do not evaluate, definite integrals which represent the given quantities. Use proper notation.

(a) The area of **R**.



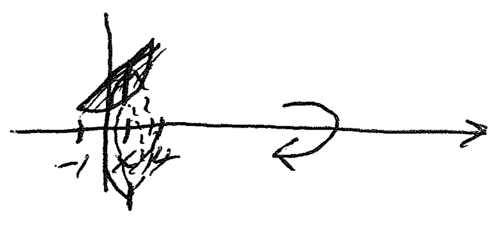
$$A = \int_{-1}^{4} \left((3x+6)-(x^2+2)\right) dx$$

(b) The volume of the solid obtained when **R** is revolved around the vertical line $x = 10$.



$$V = \int_{-1}^{4} 2\pi \underbrace{(10-x)}_{\text{rad.}} \underbrace{\left((3x+6)-(x^2+2)\right)}_{\text{height}} dx$$

(c) The volume of the solid obtained when **R** is revolved around the x-axis.



$$V = \int_{-1}^{4} \left(\pi(3x+6)^2 - \pi(x^2+2)^2\right) dx$$