(circle your TA discussion section)

- AD1, TR 1:00-1:50, Sarah Son
- AD4, TR 1:00-1:50, Sogol Jahanbekam
- AD7, TR 3:00-3:50, Nersés Aramyan
- AD9, MW 9:00-10:50, Ben Reiniger

- AD2, TR 1:00-1:50, Daniel Hockensmith
- AD5, TR 2:00-2:50, Daniel Hockensmith
- AD8, MW 11:00-12:50, Austin Rochford

- Sit in your assigned seat (shown below).
- Do not open this test booklet until I say START.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- You must show sufficient work to justify each answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Quit working and close this test booklet when I say STOP.
- Quickly turn in your test to me or a TA and show your Student ID.
1. (9 points) Find \( h'(t) \) given that \( h(t) = 40t^3 + \frac{1}{3\sqrt{t}} - 18 \)

\[
h(t) = 40t^3 + \frac{1}{3} t^{-1/2} - 18
\]

\[
h'(t) = 120t^2 - \frac{1}{6} t^{-3/2}
\]

2. (9 points) Find \( \frac{dq}{dt} \) given that \( q = 5t^2 \sec t \)

\[
\frac{dq}{dt} = (5t^2)'(\sec t) + (5t^2)(\sec t)'
\]

\[
= 10t \sec t + 5t^2 \sec t \tan t
\]

3. (9 points) Find \( f'(x) \) given that \( f(x) = \frac{x^5}{\ln x} \)

\[
f'(x) = \frac{(x^5)'(\ln x) - (x^5)(\ln x)'}{(\ln x)^2}
\]

\[
= \frac{5x^4 \ln x - x^5 \cdot \frac{1}{x}}{(\ln x)^2}
\]

\[
= \frac{5x^4 \ln x - x^4}{(\ln x)^2}
\]
4. (9 points) Find $w'(t)$ given that $w(t) = \tan^{-1} (5t^2)$

$$w'(t) = \frac{1}{1 + (5t^2)^2} \cdot (10t)$$

$$= \frac{10t}{1 + 25t^4}$$

5. (9 points) Find $\frac{dy}{dx}$ given that $\sin(x^2 + y^3) = 5y + 8x$. It is okay to leave your answer in terms of both $x$ and $y$.

$$\frac{d}{dx} (\sin(x^2 + y^3)) = \frac{d}{dx} (5y + 8x)$$

$$\cos(x^2 + y^3) \cdot (2x + 3y^2 \frac{dy}{dx}) = 5 \frac{dy}{dx} + 8$$

$$2 \cos(x^2 + y^3) + 3y^2 \cos(x^2 + y^3) \frac{dy}{dx} = 5 \frac{dy}{dx} + 8$$

$$3y^2 \cos(x^2 + y^3) - 5 \frac{dy}{dx} = 8 - 2x \cos(x^2 + y^3)$$

$$\frac{dy}{dx} = \frac{8 - 2x \cos(x^2 + y^3)}{3y^2 \cos(x^2 + y^3) - 5}$$
6. (8 points) A poster is to contain 1000 cm² of printed matter with margins of 4 cm each at top and bottom and 2 cm at each side. Find the overall dimensions if the total area of the poster is a minimum.

\[ x + 4 \quad y + 8 \]

\[ x y = 1000 \]

\[ y = \frac{1000}{x} \]

\[ A = (x+4)(y+8) \]

\[ = (x+4)\left(\frac{1000}{x} + 8\right) \]

\[ = 1000 + 8x + \frac{4000}{x} + 32 \]

\[ = 1032 + 8x + \frac{4000}{x} \]

\[ A' = 8 - \frac{4000}{x^2} \]

\[ = \frac{8x^2 - 4000}{x^2} \]

\[ = \frac{8(x^3 - 500)}{x^2} \]

\[ A' = 0 \text{ for } x = \sqrt[3]{500} \text{ or } -\sqrt[3]{500} \]

**Values of** \( A' \)

\[ \begin{array}{c|c}
0 & 0 \\
\sqrt[3]{500} & + \\
-\sqrt[3]{500} & - \\
\end{array} \]

\[ \text{min for } x = \sqrt[3]{500} \]

**Dimensions**

\[ (x+4) \text{ by } (y+8) \]

\[ 10\sqrt[3]{500} + 4 \text{ by } 20\sqrt[3]{500} + 8 \]
7. (8 points) A particle is moving along the curve \( y = \sqrt{1 + x^3} \). As it reaches the point \((2, 3)\), the \(y\)-coordinate is increasing at a rate of 18 cm/sec. How fast is the \(x\)-coordinate of the point changing at that instant?

\[
\frac{dy}{dt} = \frac{1}{2} (1 + x^3)^{-1/2} (3x^2) \frac{dx}{dt}
\]

\[
18 = \frac{1}{2} (1 + 2^3)^{-1/2} (3(2)^2) \frac{dx}{dt}
\]

\[
18 = \frac{1}{2} (\frac{9}{2}) (12) \frac{dx}{dt}
\]

\[
\frac{dx}{dt} = 9 \text{ cm/sec}
\]

8. (8 points) Upon which interval is the graph of \( f(x) = 3x^4 - 20x^3 + 10 \) increasing?

\[
f'(x) = 12x^3 - 60x^2
\]

\[
= 12x^2(x - 5)
\]

<table>
<thead>
<tr>
<th>Values of ( f'(x) )</th>
<th>--</th>
<th>0</th>
<th>--</th>
<th>0</th>
<th>++</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>0</td>
<td>5</td>
<td>( \infty )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( f \) is increasing on interval \( [5, \infty) \)
9. (8 points) A function $f(x)$ has the following second derivative.

$$f''(x) = 8e^x (x - 6)^2 (2x - 9) (x^2 + 25)$$

Find the $x$-value for each inflection point on the graph of $f(x)$.

values of $f''(x)$

- - O + + O + +

$\begin{array}{c}
\vdots \\
4.5 \\
6
\end{array}$

inflection point at

$(4.5, f(4.5))$ since $f$

changes concavity there.

10. (8 points) The graph of a function $y = f(x)$ has a $y$-intercept of 8 and has the property that the slope of the curve at every point $P$ is twice the $y$-coordinate of $P$. What is the equation of the curve?

$$\frac{dy}{dx} = 2y \quad \quad y(0) = 8$$

$$\downarrow$$

$$y = ce^{2x}$$

$$\downarrow$$

$$y = 8e^{2x}$$
11. (5 points each) Evaluate the following limits.

(a) \( \lim_{x \to 0} \frac{1-x-e^{-x}}{x^2} \)

\[
\lim_{x \to 0} \frac{1-x-e^{-x}}{x^2} = \lim_{x \to 0} \frac{-1-x^{-x}}{2x} \rightarrow 0
\]

(b) \( \lim_{x \to \infty} \frac{\sqrt{x}}{\ln x} \rightarrow \infty \text{ quickly} \)

\( \Rightarrow \infty \text{ slowly} \)

\[
\text{or} \quad \lim_{x \to \infty} \frac{\sqrt{x}}{\ln x} = \lim_{x \to \infty} \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\frac{1}{x}} = \lim_{x \to \infty} \frac{\sqrt{x}}{2} \rightarrow \infty
\]

(c) \( \lim_{x \to \infty} (1 - \frac{1}{2x})^{3x} \)

Form \( (1)^\infty \) is indeterminate

\[
= \lim_{x \to \infty} e^{\ln((1-\frac{1}{2x})^{3x})} = \lim_{x \to \infty} e^{3x \ln(1-\frac{1}{2x})}
\]

\[
= \lim_{x \to \infty} e^{\frac{\ln(1-\frac{1}{2x})}{1/(3x)}} = e^{\lim_{x \to \infty} \frac{\ln(1-\frac{1}{2x})}{1/(3x)}} \rightarrow 0
\]

\[
= e^{\lim_{x \to \infty} \left(\frac{1}{1-\frac{1}{2x}} \cdot \frac{1/(3x)^2}{1/(3x)}\right)} = e^{\lim_{x \to \infty} \frac{-3}{2-1x}} = e^{-3/2}
\]

\[= e^{-3/2}\]
Students – do not write on this page!

1. (9 points) ______________________

2. (9 points) ______________________

3. (9 points) ______________________

4. (9 points) ______________________

5. (9 points) ______________________

6. (8 points) ______________________

7. (8 points) ______________________

8. (8 points) ______________________

9. (8 points) ______________________

10. (8 points) _____________________

11a. (5 points) ____________________

11b. (5 points) ____________________

11c. (5 points) ____________________

TOTAL (100 points) ____________