MATH 220 **Test 2** **Spring 2011**

Name SOLUTIONS

(circle your TA discussion section)

- ▷ **AD1**, TR 1:00-1:50, Sarah Son
- ▷ **AD2**, TR 1:00-1:50, Daniel Hockensmith
- ▷ **AD4**, TR 1:00-1:50, Sogol Jahanbekam
- ▷ **AD5**, TR 2:00-2:50, Daniel Hockensmith
- ▷ **AD7**, TR 3:00-3:50, Nersés Aramyan
- ▷ **AD8**, MW 11:00-12:50, Austin Rochford
- ▷ **AD9**, MW 9:00-10:50, Ben Reiniger

- Sit in your assigned seat (shown below).
- Do not open this test booklet until I say *START*.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- You must show sufficient work to justify each answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Quit working and close this test booklet when I say *STOP*.
- Quickly turn in your test to me or a TA and show your Student ID.

263	264	265	266	267	268	269	270	•	271	272	273					278	279	•	280	281	282	283	284	285	286	287
	240	241	242	243	244	245	246	•	247	248	249	250	251	252	253	254	255	•	256	257	258	259	260	261	262	
	217	218	219	220	221	222	223	•	224	225	226	227	228	229	230	231	232	•	233	234	235	236	237	238	239	
	194	195	196	197	198	199	200	•	201	202	203	204	205	206	207	208	209	•	210	211	212	213	214	215	216	
	171	172	173	174	175	176	177	•	178	179	180	181	182	183	184	185	186	•	187	188	189	190	191	192	193	
	148	149	150	151	152	153	154	•	155	156	157	158	159	160	161	162	163	•	164	165	166	167	168	169	170	
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	93	94	95	96	97	98	99	•	100	101	102	103	128	105	106	107	108	•	109	110	111	112	113	114	115	
	70	71	72	73	74	75	76	•	77	78	79	80	81	82	83	84	85	•	86	87	88	89	90	91	92	
	47	48	49	50	51	52	53	•	54	55	104	57	58	59	60	61	62	•	63	64	65	66	67	68	69	
	24	25	26	27	28	29	30	•	31	32	33	34	35	36	37	38	39	•	40	41	42	43	44	45	46	
	1	2	3	4	5	6	7	•										•	17	18	19	20	21	22	23	

FRONT OF ROOM – 314 Altgeld Hall

1. (9 points) Find $h'(t)$ given that $h(t) = 40t^3 + \frac{1}{3\sqrt{t}} - 18$

$$h(t) = 40t^3 + \tfrac{1}{3}t^{-1/2} - 18$$

$$h'(t) = 120t^2 - \tfrac{1}{6}t^{-3/2}$$

2. (9 points) Find $\frac{dq}{dt}$ given that $q = 5t^2 \sec t$

$$\frac{dq}{dt} = (5t^2)'(\sec t) + (5t^2)(\sec t)'$$

$$= 10t \sec t + 5t^2 \sec t \tan t$$

3. (9 points) Find $f'(x)$ given that $f(x) = \frac{x^5}{\ln x}$

$$f'(x) = \frac{(x^5)'(\ln x) - (x^5)(\ln x)'}{(\ln x)^2}$$

$$= \frac{5x^4 \ln x - x^5 \cdot \frac{1}{x}}{(\ln x)^2}$$

$$= \frac{5x^4 \ln x - x^4}{(\ln x)^2}$$

4. (9 points) Find $w'(t)$ given that $w(t) = \tan^{-1}\left(5t^2\right)$

$$w'(t) = \frac{1}{1+(5t^2)^2} \cdot (10t)$$

$$= \frac{10t}{1+25t^4}$$

5. (9 points) Find $\frac{dy}{dx}$ given that $\sin\left(x^2 + y^3\right) = 5y + 8x$. It is okay to leave your answer in terms of both x and y.

$$\frac{d}{dx}\left(\sin(x^2+y^3)\right) = \frac{d}{dx}(5y+8x)$$

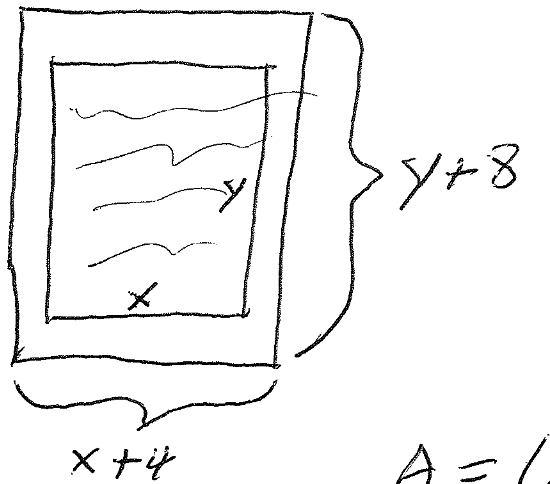
$$\cos(x^2+y^3)\cdot\left(2x+3y^2\frac{dy}{dx}\right) = 5\frac{dy}{dx}+8$$

$$2x\cos(x^2+y^3)+3y^2\cos(x^2+y^3)\frac{dy}{dx} = 5\frac{dy}{dx}+8$$

$$\left(3y^2\cos(x^2+y^3)-5\right)\frac{dy}{dx} = 8-2x\cos(x^2+y^3)$$

$$\frac{dy}{dx} = \frac{8-2x\cos(x^2+y^3)}{3y^2\cos(x^2+y^3)-5}$$

6. (8 points) A poster is to contain 1000 cm^2 of printed matter with margins of 4 cm each at top and bottom and 2 cm at each side. Find the overall dimensions if the total area of the poster is a minimum.



$xy = 1000$

$y = \frac{1000}{x}$

$A = (x+4)(y+8)$

$= (x+4)\left(\frac{1000}{x} + 8\right)$

$= 1000 + 8x + \frac{4000}{x} + 32$

$= 10\cancel{32} + 8x + \frac{4000}{x}$

$A' = 8 - \frac{4000}{x^2}$

$= \frac{8x^2 - 4000}{x^2}$

$= \frac{8(x^2 - 500)}{x^2}$

$A' = 0$ FOR $x = \sqrt{500}$ or $\cancel{-\sqrt{500}}$

values of A'

$--\ 0\ ++$

$0 \qquad \sqrt{500}$

min for $x = \sqrt{500}$

$x = \sqrt{500} = 10\sqrt{5}$

$y = \frac{1000}{\sqrt{500}} = 20\sqrt{5}$

DIMENSIONS

$(x+4)$ by $(y+8)$

$10\sqrt{5}+4$ by $20\sqrt{5}+8$

7. (8 points) A particle is moving along the curve $y = \sqrt{1 + x^3}$. As it reaches the point $(2, 3)$, the y-coordinate is increasing at a rate of 18 cm/sec. How fast is the x-coordinate of the point changing at that instant?

$$y = (1+x^3)^{1/2}$$

$$\frac{dy}{dt} = \frac{1}{2}(1+x^3)^{-1/2}(3x^2)\frac{dx}{dt}$$

$$18 = \frac{1}{2}(1+2^3)^{-1/2}(3(2)^2)\frac{dx}{dt}$$

$$18 = \frac{1}{2}\left(\frac{1}{3}\right)(12)\frac{dx}{dt}$$

$$\frac{dx}{dt} = 9 \text{ cm/sec}$$

8. (8 points) Upon which interval is the graph of $f(x) = 3x^4 - 20x^3 + 10$ increasing?

$$f'(x) = 12x^3 - 60x^2$$
$$= 12x^2(x-5)$$

values of $f'(x)$

$- -\ 0\ - -\ 0\ + +$

$0 \qquad 5 \qquad x$

f is increasing on interval

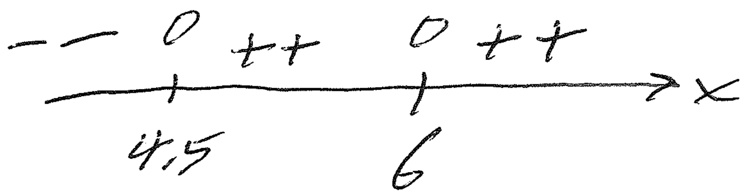
$$[5, \infty)$$

9. (8 points) A function $f(x)$ has the following second derivative.

$$f''(x) = 8e^x (x-6)^2 (2x-9) \left(x^2 + 25\right)$$

Find the x-value for each inflection point on the graph of $f(x)$.

values of $f''(x)$



inflection point at $(4.5, f(4.5))$ since f changes concavity there.

10. (8 points) The graph of a function $y = f(x)$ has a y-intercept of 8 and has the property that the slope of the curve at every point P is twice the y-coordinate of P. What is the equation of the curve?

$\frac{dy}{dx} = 2y$ $\qquad y(0) = 8$

$\downarrow$

$y = Ce^{2x}$

$\downarrow$

$y = 8e^{2x}$

11. (5 points each) Evaluate the following limits.

(a) $\lim_{x\to 0} \frac{1-x-e^{-x}}{x^2}$ $= \lim_{x\to 0} \frac{-1+e^{-x}}{2x} = \lim_{x\to 0} \frac{-e^{-x}}{2} = \frac{-1}{2}$

$\nearrow 0$, $\searrow 0$, $\nearrow 0$, $\searrow 0$

L L

(b) $\lim_{x\to\infty} \frac{\sqrt{x}}{\ln x}$ $= \infty$

$\nearrow \infty$ quickly

$\searrow \infty$ slowly

or $\lim_{x\to\infty} \frac{\sqrt{x}}{\ln x} \underset{L}{=} \lim_{x\to\infty} \frac{\frac{1}{2}x^{-1/2}}{1/x} = \lim_{x\to\infty} \frac{\sqrt{x}}{2} = \infty$

$\nearrow \infty$, $\searrow \infty$

(c) $\lim_{x\to\infty} \left(1-\frac{1}{2x}\right)^{3x}$ form $(1)^{\infty}$ is indeterminate

$= \lim_{x\to\infty} e^{\ln\left(\left(1-\frac{1}{2x}\right)^{3x}\right)} = \lim_{x\to\infty} e^{3x\ln\left(1-\frac{1}{2x}\right)}$

$= \lim_{x\to\infty} e^{\frac{\ln\left(1-\frac{1}{2x}\right)}{1/(3x)}} = e^{\lim_{x\to\infty} \frac{\ln\left(1-\frac{1}{2x}\right)}{1/(3x)}}$

$\nearrow 0$, $\searrow 0$

$\underset{L}{=} e^{\lim_{x\to\infty} \left(\frac{\frac{1}{1-\frac{1}{2x}}\cdot\left(\frac{1}{2x^2}\right)}{\left(-\frac{1}{3x^2}\right)}\right)} = e^{\lim_{x\to\infty} \frac{-3}{2-1/x}}$

$= e^{-3/2}$

Students – do not write on this page!

1. (9 points) ____________

2. (9 points) ____________

3. (9 points) ____________

4. (9 points) ____________

5. (9 points) ____________

6. (8 points) ____________

7. (8 points) ____________

8. (8 points) ____________

9. (8 points) ____________

10. (8 points) ____________

11a. (5 points) ____________

11b. (5 points) ____________

11c. (5 points) ____________

TOTAL (100 points) ____________