NAME ____________________________________________

(circle your TA discussion section)

▷ AD1, TR 1:00-1:50, Sarah Son
▷ AD4, TR 1:00-1:50, Sogol Jahanbekam
▷ AD7, TR 3:00-3:50, Nersés Aramyan
▷ AD9, MW 9:00-10:50, Ben Reiniger
▷ AD2, TR 1:00-1:50, Daniel Hockensmith
▷ AD5, TR 2:00-2:50, Daniel Hockensmith
▷ AD8, MW 11:00-12:50, Austin Rochford

• Sit in your assigned seat (shown below).
• Do not open this test booklet until I say START.
• Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
• You must show sufficient work to justify each answer.
• While the test is in progress, we will not answer questions concerning the test material.
• Quit working and close this test booklet when I say STOP.
• Quickly turn in your test to me or a TA and show your Student ID.

FRONT OF ROOM – 314 Altgeld Hall
1. (10 points) Given the function $f(x) = \frac{(\ln x)^3 - 6}{5}$, find a formula for its inverse function $f^{-1}(x)$.

2. (10 points) Find the domain of the function $f(x) = \ln \left(5 - \sqrt{12 - 4x}\right)$. 
3. (10 points) Determine the value of $k$ for which $f(x)$ is continuous throughout its domain.

$$f(x) = \begin{cases} \frac{\sin x}{x} & \text{for } x < 0 \\ 4e^x - k & \text{for } x \geq 0 \end{cases}$$

4. (5 points) Which one of the following statements is true?

(a) A function which is continuous at all points in its domain must be one-to-one.

(b) A function which is one-to-one must be increasing on its domain.

(c) A function which is continuous at a point $a$ must also be differentiable at $a$.

(d) A function which is differentiable at a point $a$ must also be continuous at $a$.

(e) A function which is differentiable at a point $a$ must be differentiable at all other points in the domain of the function.

(f) A function which is continuous at a point $a$ must be continuous at all other points in the domain of the function.
5. (12 points) Let \( f(x) = \frac{6}{x} \). Use the definition of a derivative as a limit to prove that \( f'(x) = \frac{-6}{x^2} \).

Show each step in your calculation and be sure to use proper terminology in each step of your proof.
6. (10 points) Carefully sketch a graph of \( f(x) = 3 \cos(x - \pi/2) \) on the interval \([0, 2\pi]\). Be sure to label the tick marks along the \(x\)-axis and \(y\)-axis so that the coordinates of important points on your graph are clearly shown.

7. (6 points) What is the value of \( \sin(2 \tan^{-1}(3)) \)? Write your answer as either a simplified fraction or in decimal form.

8. (7 points) Determine real numbers \(a\) and \(b\) so that the expression \( \frac{6 - 4 \sin^2 \theta}{\cos^2 \theta} \) can be rewritten as \(a \tan^2 \theta + b\).
9. (5 points each) Evaluate the following limits and simplify each answer. Show sufficient justification for each answer. An answer of ‘does not exist’ is not sufficient. If the limit is infinite then you must state if it is $\infty$ or $-\infty$.

(a) $\lim_{x \to 2} \frac{3x^2 - 2}{x^2 + 4}$

(b) $\lim_{x \to 3^+} (800 - 4 \ln (x - 3))$

(c) $\lim_{x \to 2/3} \frac{9x^2 - 4}{3x - 2}$
(d) \[ \lim_{x \to \pi/2} \frac{5 \sin^2 x}{4 \cos^2 x} \]

(e) \[ \lim_{x \to \infty} \frac{(2x + 1)^3}{6 - 5x^3} \]

(f) \[ \lim_{x \to 0} \left( \frac{2}{x} - \frac{32}{x^2 + 16x} \right) \]