

1. (10 points) Given the function $f(x) = \frac{(\ln x)^3 - 6}{5}$, find a formula for its inverse function $f^{-1}(x)$.

2. (10 points) Find the domain of the function $f(x) = \ln(5 - \sqrt{12 - 4x})$.

3. (10 points) Determine the value of k for which $f(x)$ is continuous throughout its domain.

$$f(x) = \begin{cases} \frac{\sin x}{x} & \text{for } x < 0 \\ 4e^x - k & \text{for } x \geq 0 \end{cases}$$

4. (5 points) Which one of the following statements is true?
- (a) A function which is continuous at all points in its domain must be one-to-one.
 - (b) A function which is one-to-one must be increasing on its domain.
 - (c) A function which is continuous at a point a must also be differentiable at a .
 - (d) A function which is differentiable at a point a must also be continuous at a .
 - (e) A function which is differentiable at a point a must be differentiable at all other points in the domain of the function.
 - (f) A function which is continuous at a point a must be continuous at all other points in the domain of the function.

5. (12 points) Let $f(x) = \frac{6}{x}$. Use the definition of a derivative as a limit to prove that $f'(x) = \frac{-6}{x^2}$

Show each step in your calculation and be sure to use proper terminology in each step of your proof.

6. (10 points) Carefully sketch a graph of $f(x) = 3 \cos(x - \pi/2)$ on the interval $[0, 2\pi]$. Be sure to label the tick marks along the x -axis and y -axis so that the coordinates of important points on your graph are clearly shown.

7. (6 points) What is the value of $\sin(2 \tan^{-1}(3))$? Write your answer as either a simplified fraction or in decimal form.

8. (7 points) Determine real numbers a and b so that the expression $\frac{6 - 4 \sin^2 \theta}{\cos^2 \theta}$ can be rewritten as $a \tan^2 \theta + b$.

9. (5 points each) Evaluate the following limits and simplify each answer. Show sufficient justification for each answer. An answer of 'does not exist' is not sufficient. If the limit is infinite then you must state if it is ∞ or $-\infty$.

(a) $\lim_{x \rightarrow 2} \frac{3x^2 - 2}{x^2 + 4}$

(b) $\lim_{x \rightarrow 3^+} (800 - 4 \ln(x - 3))$

(c) $\lim_{x \rightarrow 2/3} \frac{9x^2 - 4}{3x - 2}$

$$(d) \lim_{x \rightarrow \pi/2} \frac{5 \sin^2 x}{4 \cos^2 x}$$

$$(e) \lim_{x \rightarrow \infty} \frac{(2x + 1)^3}{6 - 5x^3}$$

$$(f) \lim_{x \rightarrow 0} \left(\frac{2}{x} - \frac{32}{x^2 + 16x} \right)$$

Students – do not write on this page!

1 (10 points) _____

2 (10 points) _____

3 (10 points) _____

4 (5 points) _____

5 (12 points) _____

6 (10 points) _____

7 (6 points) _____

8 (7 points) _____

9a (5 points) _____

9b (5 points) _____

9c (5 points) _____

9d (5 points) _____

9e (5 points) _____

9f (5 points) _____

TOTAL (100 points) _____