

Name SOLUTIONS

You have 13 minutes for this quiz – no calculators allowed.

1. (2 points) Find the most general antiderivative of the function $f(x) = \frac{6x^2 + 12x^4}{2x^3}$

$$f(x) = \frac{6x^2}{2x^3} + \frac{12x^4}{2x^3}$$

$$f(x) = 3 \cdot \frac{1}{x} + 6x$$

general antiderivative is

$$F(x) = 3 \ln|x| + 3x^2 + C$$

note: $F(x) = \begin{cases} 3 \ln|x| + 3x^2 + C_1 & x < 0 \\ 3 \ln|x| + 3x^2 + C_2 & x > 0 \end{cases}$ is more general

2. (2 points) Find a formula for $f(x)$ given that $f''(x) = 5e^x + 12x$, $f'(0) = 3$, and $f(0) = 15$.

$$f'(x) = 5e^x + 6x^2 + C$$

$$3 = f'(0) = 5e^0 + 6(0)^2 + C$$

$$3 = 5 + C$$

$$-2 = C$$

$$f'(x) = 5e^x + 6x^2 - 2$$

$$f(x) = 5e^x + 2x^3 - 2x + D$$

$$15 = f(0) = 5e^0 + 2(0)^3 - 2(0) + D$$

$$D = 10$$

$$f(x) = 5e^x + 2x^3 - 2x + 10$$

3. (2 points) An object is thrown upward from ground level. Three seconds later it has fallen back to the ground. What is the initial velocity for this object?

$$s'' = -9.8 \text{ m/s}^2$$

$$s' = -9.8t + C$$

$$s = -4.9t^2 + Ct + D$$

plug in $s=0$ at $t=0$ and $t=3$

$$0 = -4.9(0)^2 + C(0) + D \Rightarrow D = 0$$

$$0 = -4.9(3)^2 + C(3) + 0 \Rightarrow C = 14.7$$

$$\text{so } s' = -9.8t + 14.7$$

initial velocity is $s(0) = 14.7 \text{ m/s}$

4. (4 points) A car is traveling at 60 feet per second when the driver sees a deer in the road 300 feet ahead and immediately steps on the brakes. The deer freezes and does not move from his spot in the road. I've recorded the driver's speed (in ft/sec) every two seconds starting at the time that he first stepped on the brakes and going until the time that the car finally came to a stop. Does the car hit the deer? Explain your reasoning.

time	0	2	4	6	8
car's speed	60	46	28	12	0

assume velocity is decreasing
for $0 \leq t \leq 8$

left
Riemann
sum

$$\begin{aligned} \text{distance} &\approx 60 \cdot 2 + 46 \cdot 2 + 28 \cdot 2 + 12 \cdot 2 \\ &\approx 292 \text{ ft (overestimate)} \end{aligned}$$

right
Riemann
sum

$$\begin{aligned} \text{distance} &\approx 46 \cdot 2 + 28 \cdot 2 + 12 \cdot 2 + 0 \cdot 2 \\ &\approx 172 \text{ ft (underestimate)} \end{aligned}$$

$$172 \text{ ft} \leq \text{distance} \leq 292 \text{ ft}$$

the car does not hit the deer which was 300 ft away