Math 220 Quiz 8 (take-home) Spring 2010

Your Name Solutions

TA's Name _____

Discussion Section _____
(give either section number or meeting times)

- You may work with other students in this class. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes and the textbook.
- You should not use a calculator except to do basic arithmetic.
- Show sufficient work to justify each answer.
- The quiz should be turned in to your TA by 1pm Friday. If you will not see your TA then you can submit it to your TA's mailbox in 250 Altgeld.
- Be sure that the pages are nicely stapled – don't just fold the corners.
- Note to TA's – you should not help students with these specific problems or go over solutions until after 1pm Friday.

1. (2 points) There are currently 50 rabbits living on Lady Tottington's Estate. Suppose $\frac{dR}{dt} = 0.04R$ where R represents the number of rabbits on the estate t months from now.

(a) Find a formula for R as a function of t.

$$R = 50e^{0.04t}$$

(b) In how many months will there be 150 rabbits living on the estate?

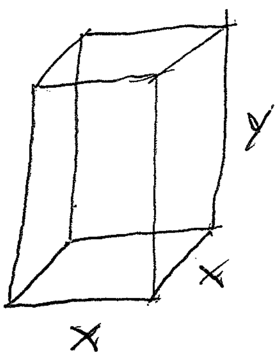
$$150 = 50e^{0.04t}$$

$$3 = e^{0.04t}$$

$$\ln 3 = 0.04t$$

$$t = \frac{\ln 3}{0.04} \approx 27.5 \text{ months}$$

2. (3 points) If a box must have a square end then what dimensions will give the box of greatest volume which can be shipped via Priority Mail? The U. S. Postal Service will accept a box for shipment via Priority Mail only if the combined length and girth (distance around) is no more than 108 inches.



$$108 = 4x + y$$

so

$$y = 108 - 4x$$

$$V = x^2 y$$

$$V = x^2(108 - 4x)$$

$$V = 108x^2 - 4x^3$$

maximize V for x in $(0, 27)$ ← $\frac{1}{4}(108)$

$$V' = 216x - 12x^2$$

$$V' = 12x(18 - x)$$

values of V'

+++ 0 − − −

0 ... 18 ... 27 → x

local max at 18.

DIMENSIONS

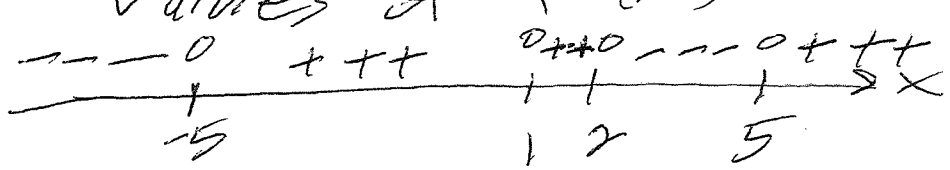
$x = 18$ in

$y = 108 - 4(18)$
$= 36$ in

3. (2 points) A function f has second derivative $f''(x) = (x-1)^4(x-2)^7(x^2-25)$. Determine the x-value for each inflection point on the graph of f.

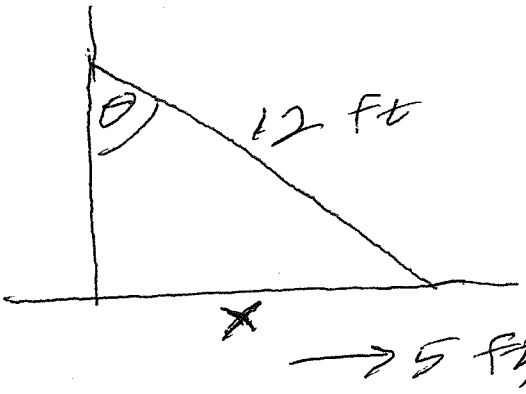
$f''(x) = (x-1)^4(x-2)^7(x-5)(x+5)$

values of $f''(x)$



* inflection points at $x = \pm 5$ and $x = 2$

4. (3 points) A ladder 12 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 0.5 feet per second, how quickly in radians per second is the angle between the ladder and the wall increasing when the bottom of the ladder is 5 feet from the wall?



given $\frac{dx}{dt} = 0.5$ ft/sec

want $\left|\frac{d\theta}{dt}\right|_{x=5\text{ ft}}$

$\sin\theta = \frac{x}{12} = \frac{1}{12}x$

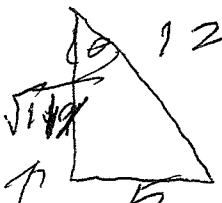
$\frac{d}{dt}(\sin\theta) = \frac{d}{dt}\left(\frac{1}{12}x\right)$

$\cos\theta \frac{d\theta}{dt} = \frac{1}{12}\frac{dx}{dt}$

$\frac{\sqrt{119}}{12} \cdot \frac{d\theta}{dt} = \frac{1}{12}(0.5)$

$\frac{d\theta}{dt} = \frac{1}{2\sqrt{119}} \approx 0.0458$ radians/sec

when $x = 5$, we have



from Pythagorean

so $\cos\theta = \frac{\sqrt{119}}{12}$