Name Solutions

- You may work with other students in this class. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes and the textbook.
- You should not use a calculator except to do basic arithmetic.
- Show sufficient work to justify each answer.
- There is no specific time limit, but the quiz is due at the beginning of Tuesday's discussion section (Monday for Merit sections).
- Note to TA's – you should not help students with these specific problems or go over solutions until the last discussion section has turned in the quiz at 3pm Tuesday.

1. (2 points) Find a formula for the derivative $\frac{dy}{dx}$ given that $x^2 + 4xy + y^2 = 13$. It is acceptable to leave your answer in terms of both x and y.

$$\frac{d}{dx}(x^2+4xy+y^2) = \frac{d}{dx}(13)$$

$$2x + \frac{d}{dx}(4xy) + 2y\frac{dy}{dx} = 0$$

$$2x + \left(4y + 4x\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$$

$$(4x+2y)\frac{dy}{dx} = -2x-4y$$

$$\frac{dy}{dx} = \frac{-2x-4y}{4x+2y} = \frac{-x-2y}{2x+y}$$

2. (2 points) Determine a formula for the line which is tangent to the graph of $y = \tan^{-1}\left(e^{6x}\right)$ at its y-intercept.

set $x=0$ to find y-intercept

$$y = \tan^{-1}(e^{6\cdot 0}) = \tan^{-1}(1)$$

$$= \pi/4$$

point $(0, \frac{\pi}{4})$

$$y' = \frac{1}{1+(e^{6x})^2} \cdot e^{6x} \cdot 6$$

$y'(0) = 3$ slope 3

$$y = 3x + \frac{\pi}{4}$$

3. (2 points) Find a formula for $f'(x)$ given that $f(x) = \ln\left(x^5 e^{x^3}\left(x^4+3\right)^6\right)$.

$f(x) = \ln(x^5) + \ln(e^{x^3}) + \ln((x^4+3)^6)$

$= 5\ln x + x^3 + 6\ln(x^4+3)$

$f'(x) = 5\cdot\frac{1}{x} + 3x^2 + 6\cdot\frac{1}{x^4+3}\cdot 4x^3$

4. (4 points) Suppose that t seconds after an object is shot directly upwards from the surface of some planet its height in feet is given by $h = 200t - 2.5t^2$.

(a) Find a formula for the velocity of the bullet at time t.

$h' = 200 - 5t$

(b) What is the maximum height attained by the bullet?

$h' = 0$ when $t = 40$

values of h'

$+++\ 0\ ---$

$\longrightarrow t$

40

max height at $t = 40$ is

$h(40) = 200(40) - 2.5(40)^2$

$= 4000$ ft