1. (2 points) Which one of the following equations must hold in order for a function f to be continuous at a number a ?

   (a) $\lim_{x\to 0} f(x) = f(a)$

   (b) $\lim_{x\to 0} f(x) = 0$

   (c) $\lim_{x\to 0} f(x) = a$

   (d) $\lim_{x\to a} f(x) = f(a)$

   (e) $\lim_{x\to a} f(x) = 0$

   (f) $\lim_{x\to a} f(x) = a$

   (g) $\lim_{x\to \infty} f(x) = f(a)$

   (h) $\lim_{x\to \infty} f(x) = 0$

   (i) $\lim_{x\to \infty} f(x) = a$

2. (2 points each) Evaluate the following limits. Show sufficient work to justify each answer.

   (a) $\lim_{x\to 0} \dfrac{\sin(2x)}{x\cos x} = \lim_{x\to 0} \dfrac{2\sin x\cos x}{x\cos x} = \lim_{x\to 0} 2\cdot\dfrac{\sin x}{x}$

   $= 2\cdot\lim_{x\to 0}\dfrac{\sin x}{x}$

   $= 2\cdot 1 = 2$

ALTERNATE APPROACH

$$\lim_{x\to 0}\frac{\sin(2x)}{x\cos x} = \lim_{x\to 0}\frac{\sin(2x)}{2x}\cdot\frac{2}{\cos x}$$

$$= \lim_{x\to 0}\frac{\sin(2x)}{2x}\cdot\lim_{x\to 0}\frac{2}{\cos x}$$

Let $u=2x$
so $u\to 0$ as $x\to 0$

$$= \lim_{u\to 0}\frac{\sin(u)}{u}\cdot\lim_{x\to 0}\frac{2}{\cos x}$$

$$= 1\cdot\frac{2}{1}$$

$$= 2$$

(b) $\lim_{x \to 3^+} \frac{8x+1}{6-2x}$ $= -\infty$

since $8x+1 \to 25$
and $6-2x \to 0^-$

(c) $\lim_{x \to \infty} \frac{6+x^2}{3+4x^2}$ $= \lim_{x \to \infty} \frac{6+x^2}{3+4x^2} \cdot \frac{1/x^2}{1/x^2}$

$= \lim_{x \to \infty} \frac{6/x^2 + 1}{3/x^2 + 4}$

$= \frac{1}{4}$

3. (2 points) Prove that $\lim_{x \to 0} 5x^6 \sin\left(\frac{3}{x}\right) = 0$.

For $x \neq 0$, we have

$-1 \leq \sin\left(\frac{3}{x}\right) \leq 1$

$-5x^6 \leq 5x^6 \sin\left(\frac{3}{x}\right) \leq 5x^6$

$\lim_{x \to 0} (-5x^6) = \lim_{x \to 0} (5x^6) = 0$

By the Squeeze Theorem,

$\lim_{x \to 0} 5x^6 \sin\left(\frac{3}{x}\right) = 0$