1. (3 points) An exponential function has a y-intercept of 4 and goes through the point $(3, 8)$. Determine a formula for this function.

x	y
0	4
3	8

$y = C \cdot a^x$

$4 = C \cdot a^0$

$4 = C$

$y = 4 \cdot a^x$

$8 = 4 \cdot a^3$

$2 = a^3$

$a = \sqrt[3]{2}$

$y = 4 \cdot (\sqrt[3]{2})^x$

equivalent answers

$y = 4 \cdot 2^{x/3}$

$y = 2^{2 + x/3}$

$y = 4e^{\frac{\ln 2}{3} x}$

2. (2 points) Determine all values of x which satisfy the equation below.

$$\ln\left(x^2 - 2x - 14\right) = 0$$

$x^2 - 2x - 14 = e^0$

$x^2 - 2x - 14 = 1$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x - 5 = 0$ or $x + 3 = 0$

$x = 5$ or $x = -3$

3. (3 points) Given that $f(x) = \sqrt[3]{2 + e^{-x}}$, find a formula for $f^{-1}(x)$.

$$\left.\begin{aligned} &\text{Let } y = f^{-1}(x) \\ &\text{then } f(y) = x \\ &\sqrt[3]{2+e^{-y}} = x \end{aligned}\right\} \text{ or just switch the roles of } x \text{ and } y$$

$$2 + e^{-y} = x^3$$

$$e^{-y} = x^3 - 2$$

$$\ln(e^{-y}) = \ln(x^3 - 2)$$

$$-y = \ln(x^3 - 2)$$

$$y = -\ln(x^3 - 2)$$

$$f^{-1}(x) = -\ln(x^3 - 2)$$

4. (2 points) Simplify the expression $\sin\left(2\cos^{-1}(3x)\right)$ into a form which does not require trigonometric or inverse trigonometric functions.

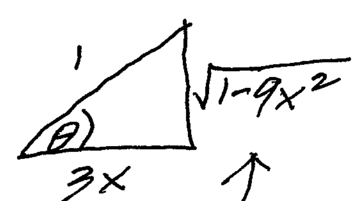
Let $\theta = \cos^{-1}(3x)$ so $\cos(\theta) = 3x$

$$\sin(2\cos^{-1}(3x)) = \sin(2\theta)$$

$$= 2\sin\theta\cos\theta$$

$$= 2\left(\frac{\sqrt{1-9x^2}}{1}\right)(3x)$$

$$= 6x\sqrt{1-9x^2}$$



Pyth. Thm