Math 220 Quiz 13 (take-home) Spring 2010

Your Name _____

TA's Name _____

solutions

Discussion Section _____
(give either section number or meeting times)

- You may work with other students in this class. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes and the textbook, but should only use a calculator to do basic arithmetic.
- The quiz should be turned in to your TA by 3pm Friday. If you will not see your TA before then you can submit it to your TA's mailbox in 250 Altgeld.
- Be sure that the pages are nicely stapled – don't just fold the corners.
- Note to TA's – you should not help students with these specific problems or go over solutions until after 3pm Friday.

Compute the following quantities. One point may be earned on each problem by simply setting up the appropriate integrals and using proper notation. For full credit you must evaluate all integrals and show sufficient work to justify your answers.

1. (3 points) The average value of $f(x) = \left(x + \frac{1}{x}\right)^2$ on the interval $[1, 3]$.

$$f_{ave} = \frac{1}{3-1}\int_1^3 \left(x+\frac{1}{x}\right)^2 dx$$

$$= \frac{1}{2}\int_1^3 \left(x^2 + 2 + \frac{1}{x^2}\right) dx$$

$$= \frac{1}{2}\left(\frac{1}{3}x^3 + 2x - \frac{1}{x}\right)\Big|_1^3$$

$$= \frac{1}{2}\left(\left(\frac{27}{3} + 6 - \frac{1}{3}\right) - \left(\frac{1}{3} + 2 - 1\right)\right)$$

$$= \frac{1}{2}\left(\frac{40}{3}\right) = \frac{20}{3} = 6.\overline{6}$$

2. (3 points) The average value of $f(x) = \sin^3 x \cos^3 x$ on the interval $\left[0, \frac{\pi}{2}\right]$.

$$f_{ave} = \frac{1}{\frac{\pi}{2} - 0} \int_0^{\pi/2} \sin^3 x \cos^3 x \, dx$$

$$= \frac{2}{\pi} \int_0^{\pi/2} \sin^3 x \cos^2 x \cos x \, dx$$

$$= \frac{2}{\pi} \int_0^{\pi/2} \sin^3 x (1 - \sin^2 x) \cos x \, dx$$

$$= \frac{2}{\pi} \int_0^1 u^3 (1 - u^2) \, du \qquad (u = \sin x, \; du = \cos x \, dx)$$

$$= \frac{2}{\pi} \int_0^1 (u^3 - u^5) \, du$$

$$= \frac{2}{\pi} \left(\frac{1}{4} u^4 - \frac{1}{6} u^6 \right) \Big|_0^1$$

$$= \frac{2}{\pi} \left(\left(\frac{1}{4} - \frac{1}{6} \right) - 0 \right) = \frac{1}{6\pi}$$

3. (4 points) The length of the curve $f(x) = \frac{1}{4}x^2 - \frac{1}{2}\ln x$ for $1 \le x \le 5$.

$$L = \int_1^5 \sqrt{1 + \left(\left(\frac{1}{4}x^2 - \frac{1}{2}\ln x \right)' \right)^2} \, dx$$

$$= \int_1^5 \sqrt{1 + \left(\frac{1}{2}x - \frac{1}{2x} \right)^2} \, dx$$

$$= \int_1^5 \sqrt{1 + \frac{x^2}{4} - \frac{1}{2} + \frac{1}{4x^2}} \, dx$$

$$= \int_1^5 \sqrt{\frac{x^2}{4} + \frac{1}{2} + \frac{1}{4x^2}} \, dx$$

$$= \int_1^5 \sqrt{\left(\frac{x}{2} + \frac{1}{2x} \right)^2} \, dx$$

$$= \int_1^5 \left(\frac{x}{2} + \frac{1}{2x} \right) dx$$

$$= \left(\frac{x^2}{4} + \frac{1}{2}\ln|x| \right) \Big|_1^5$$

$$= \left(\frac{25}{4} + \frac{1}{2}\ln 5 \right) - \left(\frac{1}{4} + \frac{1}{2}\ln 1 \right)$$

$$= 6 + \frac{1}{2}\ln 5$$