

Name solutions

You have 18 minutes for this quiz – no calculators allowed.

1. (2 points) The area between the x -axis and the graph of $f(x) = \ln x$ on the interval $[3, 7]$ can be written as a limit. Fill in the missing information in this limit.

$$AREA = \lim_{n \rightarrow \infty} \sum_{k=1}^n \left[\ln \left(3 + k \cdot \frac{4}{n} \right) \cdot \frac{4}{n} \right]$$

2. (2 points) Evaluate the following limit.

$$\begin{aligned} \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{10k}{n^2} + \frac{3}{n} \right) &= \lim_{n \rightarrow \infty} \left(\frac{10}{n^2} \sum_{k=1}^n k + \frac{3}{n} \sum_{k=1}^n 1 \right) \\ &= \lim_{n \rightarrow \infty} \left(\frac{10}{n^2} \cdot \frac{n(n+1)}{2} + \frac{3}{n} \cdot n \right) \\ &= \lim_{n \rightarrow \infty} \left(\frac{5n+5}{n} + 3 \right) \\ &= \lim_{n \rightarrow \infty} \left(5 + \frac{5}{n} + 3 \right) \\ &= 5 + 0 + 3 \\ &= \boxed{8} \end{aligned}$$

3. (2 points) Evaluate and simplify the following definite integral.

$$\begin{aligned}\int_1^2 (6x - 5) dx &= (3x^2 - 5x) \Big|_1^2 \\ &= (12 - 10) - (3 - 5) \\ &= 2 - (-2) \\ &= \boxed{4}\end{aligned}$$

4. (2 points) Evaluate the following indefinite integral.

$$\begin{aligned}\int (-4 + \tan^2 x) dx &= \int (-4 + \sec^2 x - 1) dx \\ &= \int (-5 + \sec^2 x) dx \\ &= -5x + \tan x + C\end{aligned}$$

5. (2 points) Suppose that f is integrable on the interval $[2, 9]$. Given that $\int_2^5 f(x) dx = 4$ and $\int_2^9 f(x) dx = 25$, evaluate the following definite integrals.

$$\begin{aligned}\text{(a) } \int_5^9 f(x) dx &= \int_2^9 f(x) dx - \int_2^5 f(x) dx \\ &= 25 - 4 = \boxed{21}\end{aligned}$$

$$\begin{aligned}\text{(b) } \int_5^2 f(x) dx &= -\int_2^5 f(x) dx \\ &= \boxed{-4}\end{aligned}$$