MATH 220 (sections AL1 and BL1) Test 2 Spring 2010

1. (10 points) Evaluate the following derivatives.

(a) $\frac{d}{dx}(\csc x) =$ $-\csc x \cot x$

(b) $\frac{d}{dx}(\cot x) =$ $-\csc^2 x$

(c) $\frac{d}{dx}\left(\sin^{-1} x\right) =$ $\frac{1}{\sqrt{1-x^2}}$

(d) $\frac{d}{dx}\left(\tan^{-1} x\right) =$ $\frac{1}{1+x^2}$

(e) $\frac{d}{dx}(3^x) =$ $3^x \cdot \ln 3$

2. (8 points) Find $g'(t)$ given that $g(t) = 5t^3 - 3t^2 + 15t - 18$

$$g'(t) = 15t^2 - 6t + 15$$

3. (8 points) Find $f'(x)$ given that $f(x) = \frac{\tan x}{x^3}$

$$f'(x) = \frac{\sec^2 x \cdot x^3 - \tan x \cdot 3x^2}{x^6}$$

4. (8 points) Find $P'(t)$ given that $P(t) = \sin\left(t^8 - 10t^2 + 5\right)$

$$P'(t) = \cos(t^8 - 10t^2 + 5) \cdot (8t^7 - 20t)$$

5. (5 points) Find $\frac{dy}{dx}$ given that $y = x^{\ln x}$

$$\ln y = \ln (x^{\ln x})$$

$$\ln y = \ln x \cdot \ln x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x} \cdot \ln x + \ln x \cdot \frac{1}{x} = \frac{2\ln x}{x}$$

$$\frac{dy}{dx} = y \cdot \frac{2\ln x}{x}$$

$$\frac{dy}{dx} = x^{\ln x} \cdot \frac{2\ln x}{x}$$

6. (8 points) Find $\frac{dy}{dx}$ given that $x^2y^3 = 20x + 6y$. It is okay to leave your answer in terms of both x and y.

$$2x \cdot y^3 + x^2 \cdot 3y^2 \frac{dy}{dx} = 20 + 6 \cdot \frac{dy}{dx}$$

$$(3x^2y^2 - 6)\frac{dy}{dx} = 20 - 2xy^3$$

$$\frac{dy}{dx} = \frac{20 - 2xy^3}{3x^2y^2 - 6}$$

7. (5 points) The graph of a function $y = f(x)$ has the property that the slope of the tangent line at each point on this graph is equal to one half of its y-coordinate. If the graph goes through the point $(0, 6)$, then find a formula for $f(x)$.

$$\frac{dy}{dx} = 0.5y$$

$$y = Ce^{0.5x}$$

Plugging in $(x, y) = (0, 6)$

$$6 = Ce^{0.5(0)} \Rightarrow C = 6$$

$$y = 6e^{0.5x}$$

8. (10 points) When a thin circular plate of metal is heated in an oven, its radius increases at the rate of 0.01 centimeters per minute. At what rate is the plate's area increasing when the radius is 80 centimeters?

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\left.\frac{dA}{dt}\right|_{r=80} = 2\pi(80)\cancel{(0.1)}\ 0.01$$

$$= 1.6\pi \text{ cm}^2/\text{min}$$

9. (10 points) What are the coordinates (x, y) for the highest point on the graph of the function $g(x) = 10xe^{-2x}$?

$$g'(x) = 10e^{-2x} + 10x\,e^{-2x}\cdot(-2)$$

$$g'(x) = \frac{10-20x}{e^{2x}}$$

$$g'(x) = 0 \text{ at } x = 1/2$$

values of $g'(x)$

$+ + +$ 0 $- - -$ x

$1/2$

so max attained at $x = \frac{1}{2}$

$$(x,y) = \left(\tfrac{1}{2}, 10\cdot\tfrac{1}{2}e^{-2(\frac{1}{2})}\right)$$

$$= \left(\tfrac{1}{2}, \tfrac{5}{e}\right)$$

10. (6 points) A function $f(x)$ is given below along with its first and second derivatives in factored and unfactored forms.

- $f(x) = x^4 - 4x^3 + 16x - 16 = (x+2)(x-2)^3$
- $f'(x) = 4x^3 - 12x^2 + 16 = 4(x+1)(x-2)^2$
- $f''(x) = 12x^2 - 24x = 12x(x-2)$

values of f'

$- - -$ 0 $+ + +$ 0 $+ + +$ x

-1 2

The graph of $f(x)$ is decreasing upon which one of the following intervals?

(a) $(-2, 2)$

(b) $(-1, 2)$

(c) $(0, 2)$

(d) $(-\infty, 2)$

(e) $(-\infty, -1)$

(f) $(-\infty, 0)$

(g) $(-2, \infty)$

(h) $(-1, \infty)$

(i) $(0, \infty)$

(j) $(-\infty, \infty)$

11. (12 points) Evaluate the following limits. Show sufficient work to justify each answer.

(a) $\lim_{x\to 0} \frac{\cos x}{\sin^2 x}$ $= \infty$

$\cos x \to 1$, $\sin^2 x \to 0^+$

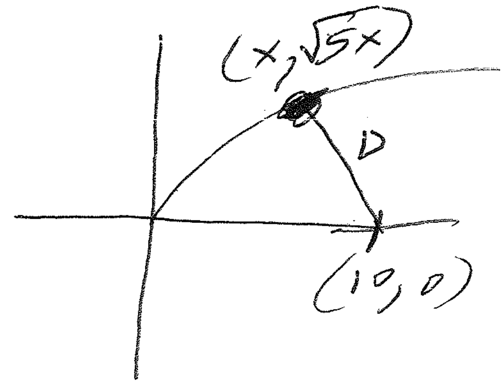
(b) $\lim_{x\to 0} \frac{e^{6x} - 6x - 1}{x^2}$ $= \lim_{x\to 0} \frac{6e^{6x} - 6}{2x} = \lim_{x\to 0} \frac{36e^{6x}}{2} = \frac{36}{2} = 18$

$e^{6x} - 6x - 1 \to 0$, $x^2 \to 0$; $6e^{6x} - 6 \to 0$, $2x \to 0$

L'Hospital's Rule

12. (8 points) Which point on the graph of $y = \sqrt{5x}$ is closest to the point $(10, 0)$?

$$D = \sqrt{(x-10)^2 + (\sqrt{5x} - 0)^2}$$



$$D = \sqrt{x^2 - 20x + 100 + 5x}$$

$$D = \sqrt{x^2 - 15x + 100}$$

$$D' = \frac{2x - 15}{2\sqrt{x^2 - 15x + 100}}$$

$D' = 0$ at $x = 7.5$

values of D'

$- - - \quad 0 \quad + + +$

$\longrightarrow x$

7.5

min at $x = 7.5$

point $(x, y) = (7.5, \sqrt{5 \cdot 7.5})$

$= (7.5, \sqrt{37.5})$

13. (2 points) Have a good Spring Break!