MATH 220 (section AL1) Test 1 Spring 2010

1. (6 points) If the point $(7, -2)$ is on the graph of an even function f, then which one of the following points must also be on the graph of f ?

(a) $(2, 7)$
(b) $(2, -7)$
(c) $(-2, 7)$
(d) $(-2, -7)$
(e) $(7, 2)$
(f) $(-7, 2)$
(g) $(-7, -2)$

f even $\Rightarrow f(-x) = f(x)$
so $f(-7) = f(7) = -2$
and $(-7, -2)$ is on the graph of f

2. (6 points) If the point $(7, -2)$ is on the graph of an odd function f, then which one of the following points must also be on the graph of f ?

(a) $(2, 7)$
(b) $(2, -7)$
(c) $(-2, 7)$
(d) $(-2, -7)$
(e) $(7, 2)$
(f) $(-7, 2)$
(g) $(-7, -2)$

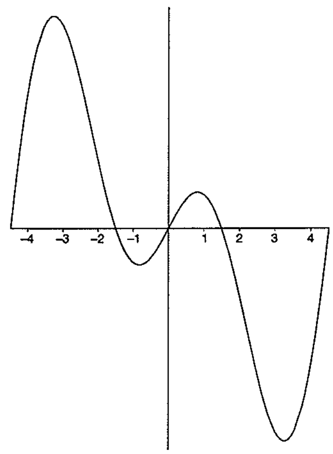
f odd $\Rightarrow f(-x) = -f(x)$
so $f(-7) = -f(7)$
$= -(-2)$
$= 2$
and $(-7, 2)$ is on the graph of f

3. (6 points) Given a function $f(x)$ for which $\lim_{h\to 0} \dfrac{f(4+h) - f(4)}{h}$ exists, which one of the following statements must be true?

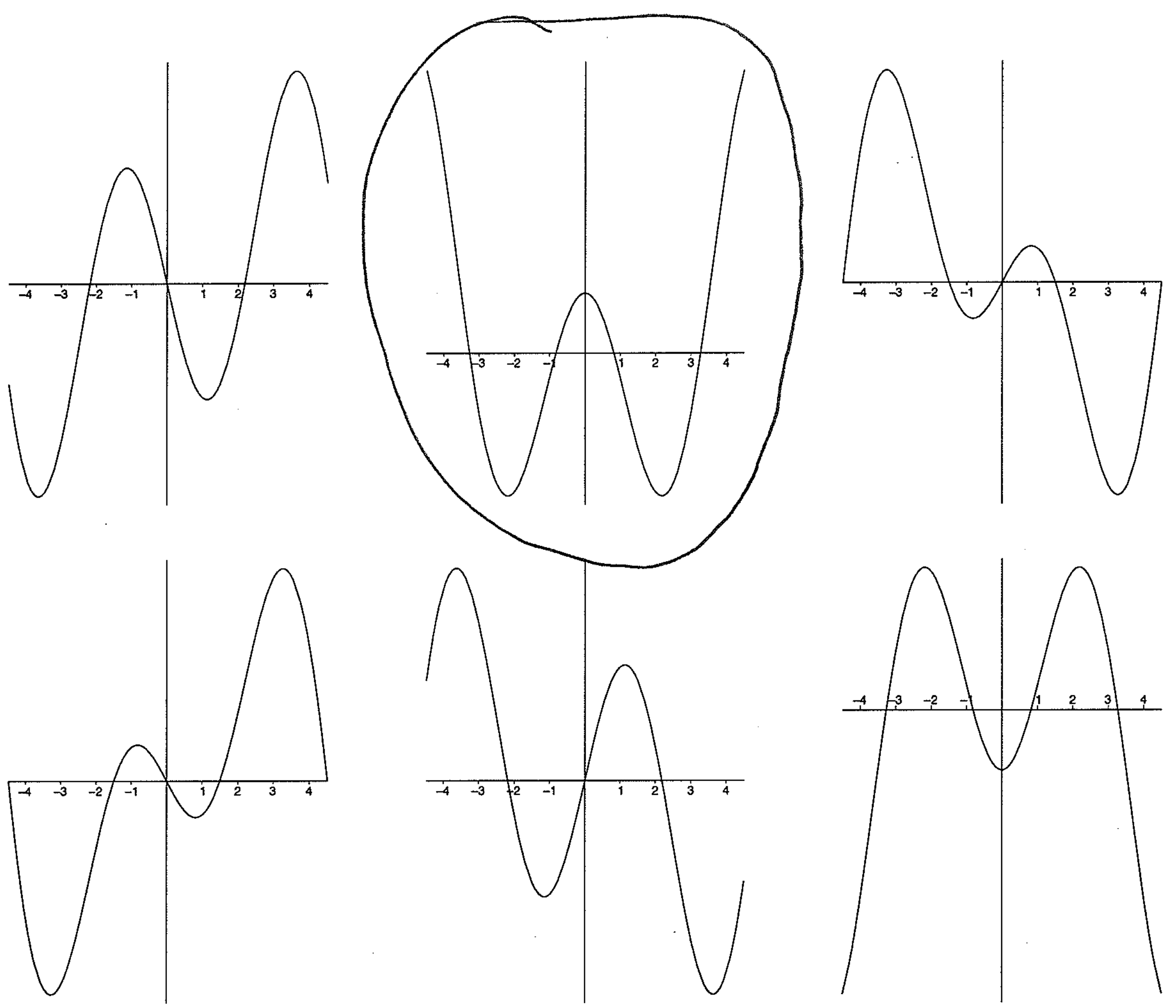
(a) f is continuous but not differentiable at $x = 0$.
(b) f is continuous but not differentiable at $x = 4$.
(c) f is differentiable but not continuous at $x = 0$.
(d) f is differentiable but not continuous at $x = 4$.
(e) f is both differentiable and continuous $x = 0$.
(f) f is both differentiable and continuous $x = 4$.
(g) f is neither continuous nor differentiable at $x = 0$.
(h) f is neither continuous nor differentiable at $x = 4$.

$f'(4) = \lim_{h \to 0} \dfrac{f(4+h) - f(4)}{h}$
This limit exists so f is differentiable at $x = 4$.
Since differentiability implies continuity, f is also continuous at $x = 4$.

4. (6 points) The graph of $f(x)$ is shown below.



Circle the graph of $f'(x)$, given that it is one of the six choices below.



5. (12 points) Given that $f(x) = 5 + \ln(x-4)$, find a formula for $f^{-1}(x)$.

Let $y = f^{-1}(x)$

Then $f(y) = x$

$5 + \ln(y-4) = x$

or just switch the roles of x and y and solve for y

$\ln(y-4) = x-5$

$y - 4 = e^{x-5}$

$y = 4 + e^{x-5}$

$f^{-1}(x) = 4 + e^{x-5}$

6. (10 points) Let $f(x) = x^2 - 6x$. Use the definition of a derivative as a limit to show that $f'(x) = 2x - 6$. Show each step in your calculation and be sure to use proper terminology.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - 6(x+h) - (x^2 - 6x)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 6x - 6h - x^2 + 6x}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - 6h}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h - 6)}{h}$$

$$= \lim_{h \to 0} (2x + h - 6)$$

$$= 2x - 6$$

ANOTHER METHOD

$$f'(x) = \lim_{w \to x} \frac{f(w) - f(x)}{w - x}$$

$$= \lim_{w \to x} \frac{w^2 - 6w - (x^2 - 6x)}{w - x}$$

$$= \lim_{w \to x} \frac{w^2 - x^2 - 6w + 6x}{w - x}$$

$$= \lim_{w \to x} \frac{(w+x)(w-x) - 6(w-x)}{w - x}$$

$$= \lim_{w \to x} \frac{(w-x)(w+x-6)}{w - x} = \lim_{w \to x} (w + x - 6) = 2x - 6$$

7. (12 points) Find the domain of the function $f(x) = \ln\left(5 - \sqrt{x - 30}\right)$.

without delving into complex numbers, we can only take the square root of nonnegative numbers and the logarithm of positive numbers. Thus

$x - 30 \geq 0$ and $5 - \sqrt{x-30} > 0$

$x \geq 30$ and $5 > \sqrt{x-30}$

$25 > x - 30$

$55 > x$

comment

$5 > -10$ but 5^2 (or 25) is not greater than $(-10)^2$ (or 100)

on the test why was it okay to square both sides

Domain: $[30, 55)$

8. (12 points) Find a formula for an exponential function whose graph goes through the following three points.

$$(0, 9),\ (3, 12),\ (6, 16)$$

$y = Ca^x$

$9 = Ca^0 = C$

so $y = 9a^x$

$12 = 9a^3$

$a^3 = \frac{12}{9} = \frac{4}{3}$

$a = \left(\frac{4}{3}\right)^{1/3}$

so $y = 9\left(\left(\frac{4}{3}\right)^{1/3}\right)^x = 9\left(\frac{4}{3}\right)^{x/3}$

CHECK

x	$y = 9\left(\frac{4}{3}\right)^{x/3}$	
0	9	✓
3	12	✓
6	16	✓

9. (5 points each) Evaluate the following limits. An answer of 'does not exist' is not sufficient. If the limit is infinite then you must state if it is ∞ or $-\infty$.

(a) $\lim_{x\to 0}\left(18 - 11\ln\left(5x^2+1\right)\right)$ $= 18 - 11\cdot\ln(5\cdot 0^2+1)$

$= 18 - 11\cdot\ln(1)$

$= 18 - 11\cdot 0$

$= 18$

(b) $\lim_{x\to\infty}\left(9 + 8\cos\left(e^{-3x}\right)\right)$ $= \lim_{x\to\infty}\left(9 + 8\cdot\cos\left(\frac{1}{e^{3x}}\right)\right)$

$= 9 + 8\cdot\cos(0)$

$= 9 + 8\cdot 1$

$= 17$

(c) $\lim_{x\to 5}\frac{x^2+2x-35}{x-5}$ $= \lim_{x\to 5}\frac{(x-5)(x+7)}{x-5}$

$= \lim_{x\to 5}(x+7)$

$= 12$

(d) $\lim_{x\to 4^-} \frac{3x^2+10}{x^2-16}$ $\xrightarrow{} 58$, $\xrightarrow{} 0^-$ $= -\infty$

(e) $\lim_{x\to\infty} \frac{3x+5x^2}{7x^2+13} = \lim_{x\to\infty} \frac{(3x+5x^2)\cdot\frac{1}{x^2}}{(7x^2+13)\cdot\frac{1}{x^2}}$

$= \lim_{x\to\infty} \frac{\frac{3}{x}+5}{7+\frac{13}{x^2}} = \frac{0+5}{7+0} = \frac{5}{7}$

10. (5 points) A function f satisfies the following inequality for all $x \neq 0$.

$$\frac{9x+2\sin x}{2x} \leq f(x) \leq \frac{13x-2\sin x}{2x}$$

Determine $\lim_{x\to 0} f(x)$.

$\lim_{x\to 0} \frac{9x+2\sin x}{2x} = \lim_{x\to 0}\left(\frac{9x}{2x} + \frac{2\sin x}{2x}\right)$

$= \lim_{x\to 0}\left(\frac{9}{2} + \frac{\sin x}{x}\right)$

$= \frac{9}{2} + 1 = \frac{11}{2}$

$\lim_{x\to 0} \frac{13x-2\sin x}{2x} = \lim_{x\to 0}\left(\frac{13x}{2x} - \frac{2\sin x}{2x}\right)$

$= \lim_{x\to 0}\left(\frac{13}{2} - \frac{\sin x}{x}\right)$

$= \frac{13}{2} - 1 = \frac{11}{2}$

By the ~~Sq~~ Squeeze Theorem, $\lim_{x\to 0} f(x) = \frac{11}{2}$