Math 220 – Test 3 Information – Sections AL1, AL2 and AL3 with Bob Murphy

The test will be given during your lecture period on Wednesday (December 4, 2019). No books, notes, scratch paper, calculators or other electronic devices are allowed. Bring a Student ID.

It may be helpful to look at

- [https://faculty.math.illinois.edu/~murphyrf/teaching/M220-F2019/](https://faculty.math.illinois.edu/~murphyrf/teaching/M220-F2019/) – All handouts and worksheets since the one on properties of definite integrals, quizzes 8, 9, 10, 11 and 12, and Daily Assignments for a summary of each lecture

- [https://compass2g.illinois.edu/](https://compass2g.illinois.edu/) – Homework solutions

- [https://faculty.math.illinois.edu/~murphyrf/teaching/M220/](https://faculty.math.illinois.edu/~murphyrf/teaching/M220/) – Tests and quizzes in my previous MATH 220 courses

- **Section 3.10 (Linear Approximation and Differentials)**
  - Be able to use a tangent line (or differentials) in order to approximate the value of a function near the point of tangency.

- **Section 4.2 (The Mean Value Theorem)**
  - Be able to precisely state The Mean Value Theorem and Rolle’s Theorem.
  - For test 3 and the final exam, you will not have to prove anything about the number of roots for a function.

- **Section 4.8 (Newton’s Method)**
  - Understand the graphical basis for Newton’s Method (that is, use the point where the tangent line crosses the x-axis as your next estimate for a root of a function).
  - Be able to apply Newton’s Method to approximate roots, solutions, x-intercepts, intersection points, critical numbers, inflection points, points where the line tangent or normal to the curve is parallel or perpendicular to some other line, etc.

- **Section 4.9 (Antiderivatives)**
  - Know antiderivative formulas for 0, k (a constant), sin x, cos x, sec^2 x, csc^2 x, sec x tan x, csc x cot x, e^x, a^x, \( \frac{1}{1 + x^2} \), \( \frac{1}{\sqrt{1 - x^2}} \), \( \frac{1}{x\sqrt{x^2 - 1}} \), \( x^n \) (n ≠ -1), \( x^{-1} = \frac{1}{x} \).
  - Be able to find general antiderivatives for functions which are sums or differences of constants multiplied by the above formulas (you may need to simplify first).
  - Be able to solve a differential equation where values for the function or its first or second derivative are given.
  - Be able to apply these rules to problems involving acceleration, velocity, or position.
• **Section 5.1 (Areas and Distances)**
  
  – Use Riemann sums (left, right, or midpoint) to estimate area or total change in a quantity and state if your estimate is known to be an underestimate or overestimate. These sums will involve at most 8 subintervals.
  
  – Use limits of Riemann sums to find the exact area or total change in a quantity. Being able to do this with right Riemann sums will be sufficient for this test.
  
  – Understand sigma notation for sums and know the following sums.
    
    * \[ \sum_{k=1}^{n} C = C \cdot n \] (C is a constant)
    
    * \[ \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \]
    
    * \[ \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \]
    
    * \[ \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2 \]
  
  – Be able to take the limit as \( n \to \infty \) for sums including the types listed above.

• **Section 5.2 (The Definite Integral)**
  
  – Understand the definition of a definite integral as \[ \int_{a}^{b} f(x) \, dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*) \Delta x. \]
  
  – Be able to write area, change in position, or the value of a definite integral as the limit of a Riemann sum. Being able to write this limit with a right Riemann sum will be sufficient for this test.
  
  – Be able to evaluate the limit as \( n \to \infty \) for a Riemann sum.
  
  – Know the relationship between a definite integral and area. This should be understood regardless of whether or not the graph of the function being integrated is above or below the \( x \)-axis.
  
  – Know the following properties of the definite integral.
    
    * \[ \int_{b}^{a} f(x) \, dx = -\int_{a}^{b} f(x) \, dx \]
    
    * \[ \int_{a}^{a} f(x) \, dx = 0 \]
    
    * \[ \int_{a}^{b} c \, dx = c(b-a) \] where \( c \) is any constant
    
    * \[ \int_{a}^{b} [f(x) + g(x)] \, dx = \int_{a}^{b} f(x) \, dx + \int_{a}^{b} g(x) \, dx \]
    
    * \[ \int_{a}^{b} c f(x) \, dx = c \int_{a}^{b} f(x) \, dx \] where \( c \) is any constant
    
    * \[ \int_{a}^{b} [f(x) - g(x)] \, dx = \int_{a}^{b} f(x) \, dx - \int_{a}^{b} g(x) \, dx \]
    
    * \[ \int_{c}^{c} f(x) \, dx + \int_{c}^{b} f(x) \, dx = \int_{a}^{b} f(x) \, dx \]
\* If \( f(x) \geq 0 \) for \( a \leq x \leq b \), then \( \int_a^b f(x) \, dx \geq 0 \)
\* If \( f(x) \geq g(x) \) for \( a \leq x \leq b \), then \( \int_a^b f(x) \, dx \geq \int_a^b g(x) \, dx \)
\* If \( m \leq f(x) \leq M \) for \( a \leq x \leq b \), then \( m(b-a) \leq \int_a^b f(x) \, dx \leq M(b-a) \)

- **Section 5.3 (The Fundamental Theorem of Calculus)**
  - Be able to precisely state Part 1 and Part 2 of the The Fundamental Theorem of Calculus.
  - When the conditions of the theorem hold, be able to use Part 1 to find the derivative of functions which are defined in terms of integrals.
  - When the conditions of the theorem hold, be able to use Part 2 to evaluate definite integrals.

- **Section 5.4 (Indefinite Integrals and the Net Change Theorem)**
  - Know indefinite integral formulas for 0, \( k \) (a constant), \( \sin x, \cos x, \sec^2 x, \csc^2 x, \sec x \tan x, \csc x \cot x, e^x, a^x, \frac{1}{1+x^2}, \frac{1}{\sqrt{1-x^2}}, \frac{1}{x\sqrt{x^2-1}}, x^n \) \((n \neq -1)\), \( x^{-1} = \frac{1}{x} \).
  - Know that the definite integral of a rate of change gives the total change. Be able to use this Net Change Theorem for applied problems involving rates of change such as velocity, acceleration, growth rates, etc.

- **Section 5.5 (The Substitution Rule)**
  - Be able to solve a wide variety of definite or indefinite integrals using substitution.
  - Be able to more quickly evaluate definite integrals on the interval \([-a,a]\) when the integrand is an even function or an odd function.

- **Section 6.1 (Areas between Curves)**
  - Be able to find areas between curves. This may require breaking the area up into the sum of two or more definite integrals.
  - Be able to integrate with respect to \( x \) or with respect to \( y \) to determine these areas.

- **Section 6.2 (Volumes)**
  - Be able to find volumes for solids formed by revolving a region around any vertical or horizontal line.
  - Be able to find volumes for solids formed by building upon some base and having cross-sections which are semi-circles, squares, rectangles, triangles, etc.
  - Be able to integrate with respect to \( x \) or with respect to \( y \) to determine these volumes.
• Section 6.3 (Volumes by Cylindrical Shells)
  – Be able to find volumes for solids formed by revolving a region around any vertical or horizontal line.
  – Be able to integrate with respect to $x$ or with respect to $y$ to determine these volumes.

• Section 6.5 (Average Value of a Function)
  – Be able to find the average value of a function.
  – Know the graphical interpretation of the average value of a function.

• Section 7.2 (Trigonometric Integrals)
  – Be able to use substitution to solve definite or indefinite integrals involving trigonometric functions.
  – Be able to evaluate trigonometric functions at special angles.
  – To evaluate some integrals, you should know the following trigonometric definitions and identities.

\[
\begin{align*}
* \tan x &= \frac{\sin x}{\cos x} \\
* \cot x &= \frac{\cos x}{\sin x} \\
* \sec x &= \frac{1}{\cos x} \\
* \csc x &= \frac{1}{\sin x} \\
* \sin^2 x + \cos^2 x &= 1 \\
* \tan^2 x + 1 &= \sec^2 x \\
* 1 + \cot^2 x &= \csc^2 x \\
* \sin (2x) &= 2 \sin x \cos x \\
* \cos (2x) &= \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x \\
* \cos^2 x &= \frac{1}{2} + \frac{1}{2} \cos (2x) \\
* \sin^2 x &= \frac{1}{2} - \frac{1}{2} \cos (2x)
\end{align*}
\]