Math 220 (section AD?) Quiz 9 Fall 2019

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points each) Evaluate the following indefinite integrals.

(a) $\int (3x^2-4)\left(\frac{2}{x^2+1}+9\right)\,dx =$

$$\int\left(\frac{6x^2}{x^2+1}+27x^2-\frac{8}{x^2+1}-36\right)dx =$$

$$\int\left(\frac{6x^2+6-6}{x^2+1}+27x^2-\frac{8}{x^2+1}-36\right)dx =$$

$$\int\left(\frac{6(x^2+1)-6}{x^2+1}+27x^2-\frac{8}{x^2+1}-36\right)dx =$$

$$\int\left(6-\frac{6}{x^2+1}+27x^2-\frac{8}{x^2+1}-36\right)dx =$$

$$\int\left(27x^2-\frac{14}{x^2+1}-30\right)dx = 9x^3-14\arctan(x)-30x+C$$

(b) $\int \frac{\cos(2x)-2\cos^2(x)+6}{\cos(x)\cot(x)}\,dx =$

$$\int\frac{(2\cos^2(x)-1)-2\cos^2(x)+6}{\cos(x)\cot(x)}\,dx =$$

$$\int\frac{5}{\cos(x)\cot(x)}\,dx =$$

$$\int 5\sec(x)\tan(x)\,dx =$$

$$5\sec(x)+C$$

2. (2 points) Evaluate the following definite integral. Simplify your answer.

$$\int_1^{e^6} \left(\frac{2}{\sqrt[3]{x^2}} + \frac{8}{x} \right) dx = \int_1^{e^6} \left(2x^{-2/3} + \frac{8}{x}\right) dx$$

$$= \left[6x^{1/3} + 8\ln|x|\right]\Big|_1^{e^6}$$

$$= \left[6(e^6)^{1/3} + 8\ln(e^6)\right] - \left[6(1)^{1/3} + 8\ln(1)\right]$$

$$= \left[6e^2 + 8\cdot 6\right] - \left[6 + 8\cdot 0\right]$$

$$= 6e^2 + 42$$

3. (2 points) At time t hours, a bacteria population grows at a rate of $100t + 80$ bacteria per hour. If the population is 200 at time $t = 1$, then what is the population at time $t = 3$ hours?

method 1) From the net change theorem,

(pop at $t=3$) = (pop at $t=1$) + (net change in pop from $t=1$ to $t=3$)

$$= 200 + \int_1^3 (\text{rate of change in pop})\, dt$$

$$= 200 + \int_1^3 (100t + 80)\, dt$$

$$= 200 + \left[50t^2 + 80t\right]\Big|_1^3$$

$$= 200 + \left[50(3)^2 + 80(3)\right] - \left[50(1)^2 + 80(1)\right]$$

$$= 200 + 690 - 130$$

$$= 760 \text{ bacteria}$$

For method 2, let

$P'(t) = 100t + 80$

$P(t) = 50t^2 + 80t + C$

$P(1) = 200 \Rightarrow C = 70$

$P(t) = 50t^2 + 80t + 70$

$P(3) = 50(3)^2 + 80(3) + 70$

$= 760$ bacteria

4. (2 points) Let $g(x) = \int_{x^4-32x}^{3} \frac{1}{t^{12}+4}\, dt$. Find the x-coordinate for the highest point on the graph of $g(x)$.

$$g(x) = -\int_{3}^{x^4-32x} \frac{1}{t^{12}+4}\, dt$$

$$g'(x) = -\frac{1}{(x^4-32x)^{12}+4} \cdot \frac{d}{dx}(x^4-32x)$$ (by part 1 of F.T.C. and the chain rule)

$$= -\frac{1}{(x^4-32x)^{12}+4} \cdot (4x^3-32)$$

$$= \frac{-4(x^3-8)}{(x^4-32x)^{12}+4}$$

$g'(x)$ exists for all x since denominator > 0

$g'(x) = 0$ when $x^3 - 8 = 0 \Rightarrow \boxed{x = 2}$

values of $g'(x)$

$+++ \quad 0 \quad ---$

$\longleftarrow\!\!\!\overset{}{|}\!\!\!\longrightarrow x$

2

g is increasing on $(-\infty, 2)$

g is decreasing on $(2, \infty)$

The highest point (i.e. abs. max) for $g(x)$ occurs when $x = 2$