

Name

Solutions

• You have 20 minutes

• No calculators

• Show sufficient work

1. (2 points each) Evaluate the following indefinite integrals.

$$(a) \int (3x^2 - 4) \left(\frac{2}{x^2 + 1} + 9 \right) dx =$$

$$\int \left(\frac{6x^2}{x^2 + 1} + 27x^2 - \frac{8}{x^2 + 1} - 36 \right) dx =$$

$$\int \left(\frac{6x^2 + 6 - 6}{x^2 + 1} + 27x^2 - \frac{8}{x^2 + 1} - 36 \right) dx =$$

$$\int \left(\frac{6(x^2 + 1) - 6}{x^2 + 1} + 27x^2 - \frac{8}{x^2 + 1} - 36 \right) dx =$$

$$\int \left(6 - \frac{6}{x^2 + 1} + 27x^2 - \frac{8}{x^2 + 1} - 36 \right) dx =$$

$$\int \left(27x^2 - \frac{14}{x^2 + 1} - 30 \right) dx = 9x^3 - 14 \arctan(x) - 30x + C$$

$$(b) \int \frac{\cos(2x) - 2\cos^2(x) + 6}{\cos(x)\cot(x)} dx =$$

$$\int \frac{(2\cos^2(x) - 1) - 2\cos^2(x) + 6}{\cos(x)\cot(x)} dx =$$

$$\int \frac{5}{\cos(x)\cot(x)} dx =$$

$$\int 5 \sec(x) \tan(x) dx =$$

$$5 \sec(x) + C$$

2. (2 points) Evaluate the following definite integral. Simplify your answer.

$$\begin{aligned}
 \int_1^{e^6} \left(\frac{2}{\sqrt[3]{x^2}} + \frac{8}{x} \right) dx &= \int_1^{e^6} \left(2x^{-2/3} + \frac{8}{x} \right) dx \\
 &= \left[6x^{1/3} + 8 \ln|x| \right]_1^{e^6} \\
 &= \left[6(e^6)^{1/3} + 8 \ln(e^6) \right] - \left[6(1)^{1/3} + 8 \ln(1) \right] \\
 &= \left[6e^2 + 8 \cdot 6 \right] - \left[6 + 8 \cdot 0 \right] \\
 &= \boxed{6e^2 + 42}
 \end{aligned}$$

3. (2 points) At time t hours, a bacteria population grows at a rate of $100t + 80$ bacteria per hour. If the population is 200 at time $t = 1$, then what is the population at time $t = 3$ hours?

method 1) From the net change theorem,

$$(\text{pop. at } t=3) = (\text{pop. at } t=1) + \left(\begin{array}{l} \text{net} \\ \text{change in pop.} \\ \text{from } t=1 \text{ to } t=3 \end{array} \right)$$

$$= 200 + \int_1^3 \left(\begin{array}{l} \text{rate of change} \\ \text{in pop.} \end{array} \right) dt$$

$$= 200 + \int_1^3 (100t + 80) dt$$

$$= 200 + \left[50t^2 + 80t \right]_1^3$$

$$= 200 + \left[50(3)^2 + 80(3) \right] - \left[50(1)^2 + 80(1) \right]$$

$$= 200 + 690 - 130$$

$$= \boxed{760 \text{ bacteria}}$$

For method 2, let

$$P'(t) = 100t + 80$$

$$P(t) = 50t^2 + 80t + C$$

$$P(1) = 200 \Rightarrow C = 70$$

$$P(t) = 50t^2 + 80t + 70$$

$$P(3) = 50(3)^2 + 80(3) + 70$$

$$= \boxed{760 \text{ bacteria}}$$

4. (2 points) Let $g(x) = \int_{x^4-32x}^3 \frac{1}{t^{12}+4} dt$. Find the x -coordinate for the highest point on the graph of $g(x)$.

$$g(x) = - \int_3^{x^4-32x} \frac{1}{t^{12}+4} dt$$

$$g'(x) = - \frac{1}{(x^4-32x)^{12}+4} \cdot \frac{d}{dx}(x^4-32x)$$

(by part 1 of F.T.C., and the chain rule)

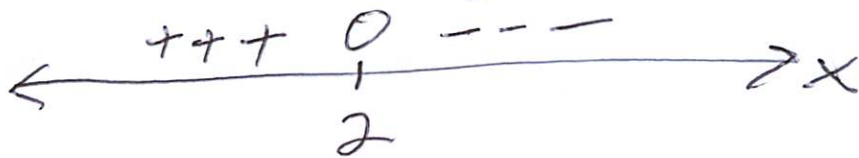
$$= - \frac{1}{(x^4-32x)^{12}+4} \cdot (4x^3-32)$$

$$= \frac{-4(x^3-8)}{(x^4-32x)^{12}+4}$$

$g'(x)$ exists for all x since denominator > 0

$$g'(x) = 0 \text{ when } x^3-8=0 \Rightarrow \boxed{x=2}$$

values of $g'(x)$



g is increasing on $(-\infty, 2)$

g is decreasing on $(2, \infty)$

The highest point (i.e. abs. max) for $g(x)$ occurs when $x=2$