Math 220 **Quiz 7 (take-home)** **Fall 2019**

Name Solutions

(circle your TA discussion section)

- ▷ **AD1**, TR 11:00-12:50, Mina Nahvi
- ▷ **AD2**, TR 9:00-10:50, Adriana Morales
- ▷ **AD3**, TR 1:00-2:50, Vincent Villalobos
- ▷ **AD@**, TR 9:00-9:50, Phuong "Sophie" Le
- ▷ **ADA**, TR 8:00-8:50, Scott Harman
- ▷ **ADB**, TR 9:00-9:50, Lutian Zhao
- ▷ **ADC**, TR 10:00-10:50, Lutian Zhao
- ▷ **ADD**, TR 11:00-11:50, Dara Zirlin
- ▷ **ADE**, TR 12:00-12:50, David Altizio
- ▷ **ADF**, TR 1:00-1:50, Saaber Pourmotabbed
- ▷ **ADG**, TR 2:00-2:50, John "Connor" Grady
- ▷ **ADH**, TR 3:00-3:50, Sarah Simpson
- ▷ **ADI**, TR 4:00-4:50, Ryan McConnell
- ▷ **ADJ**, TR 9:00-9:50, Robert "Bob" Krueger
- ▷ **ADK**, TR 10:00-10:50, Sarah Simpson
- ▷ **ADL**, TR 11:00-11:50, Rocco Davino
- ▷ **ADM**, TR 12:00-12:50, Dara Zirlin
- ▷ **ADN**, TR 1:00-1:50, John "Connor" Grady
- ▷ **ADO**, TR 2:00-2:50, Shuyu "Sonya" Xiao
- ▷ **ADQ**, TR 10:00-10:50, Saaber Pourmotabbed
- ▷ **ADR**, TR 9:00-9:50, Scott Harman
- ▷ **ADS**, TR 12:00-12:50, Rocco Davino
- ▷ **ADT**, TR 2:00-2:50, Ryan McConnell
- ▷ **ADU**, TR 3:00-3:50, Shuyu "Sonya" Xiao
- ▷ **ADW**, TR 8:00-8:50, Robert "Bob" Krueger

- You may lose points if you do not circle your correct discussion section.
- You may work with other MATH 220 students. However each student should write their solutions separately and independently – nobody should copy someone else's work.
- You may use your notes, the textbook, or information found on my course home page including old test and quiz solutions.
- You are not allowed to use a calculator, Wolfram Alpha, or any similar technology.
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **The quiz is due at the beginning of your lecture period on Friday, October 18th.**
- **TAs and Tutors – Do not help students with these specific problems until the quizzes have been collected for all MATH 220 lectures (10-10:50am, 1-1:50pm, 3-3:50pm).**

1. (3 points) Evaluate the following limit.

$$\underbrace{\lim_{x\to\infty} \left(2x^{-5}+1\right)^{4x^5}}_{\substack{\text{indeterminate}\\ \text{form } 1^\infty}} = \lim_{x\to\infty} e^{\ln\left(\left(2x^{-5}+1\right)^{4x^5}\right)}$$

$$= \lim_{x\to\infty} e^{4x^5 \ln\left(2x^{-5}+1\right)}$$

$$= \lim_{x\to\infty} e^{\frac{4\ln\left(2x^{-5}+1\right)}{x^{-5}}}$$

$$= e^{\lim_{x\to\infty} \frac{4\ln\left(2x^{-5}+1\right)}{x^{-5}}} \quad \begin{matrix} \to 0 \\ \to 0 \end{matrix}$$

$$\overset{H}{=} e^{\lim_{x\to\infty} \frac{\frac{d}{dx}\left(4\ln\left(2x^{-5}+1\right)\right)}{\frac{d}{dx}\left(x^{-5}\right)}} \quad \text{(by l'Hospital's Rule)}$$

$$= e^{\lim_{x\to\infty} \frac{\frac{4}{2x^{-5}+1}\cdot -10x^{-6}}{-5x^{-6}}}$$

$$= e^{\lim_{x\to\infty} \frac{8}{2x^{-5}+1}}$$

$$= e^{\frac{8}{2\cdot 0+1}}$$

$$= e^{8}$$

2. (3 points) Find each interval on which f is increasing and each interval on which f is decreasing.

$f(x) = 12 \arctan(5x) - 6 \ln(25x^2 + 1)$

$$f'(x) = 12 \cdot \frac{1}{(5x)^2+1} \cdot 5 - 6 \cdot \frac{1}{25x^2+1} \cdot 50x$$

$$f'(x) = \frac{60 - 300x}{25x^2+1}$$

$$f'(x) = \frac{60(1-5x)}{25x^2+1}$$

$f'(x)$ exists for all x

$f'(x) = 0 \Rightarrow 1 - 5x = 0 \Rightarrow x = \frac{1}{5}$ (critical number)

values of $f'(x)$

$+ + + \quad 0 \quad - - -$

x

$1/5$

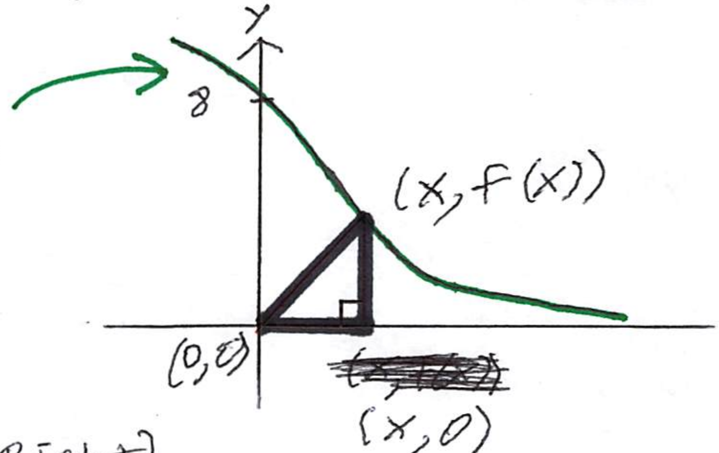
f is increasing on $(-\infty, \frac{1}{5})$

f is decreasing on $(\frac{1}{5}, \infty)$

Note: It is okay to also include $\frac{1}{5}$ for each interval

3. (4 points) For each $x > 0$, a triangle is formed with vertices $(0,0)$, $(x,0)$ and $(x, f(x))$ where $f(x)$ is the function given below. What is the value of x which results in the triangle of largest area?

$$f(x) = \frac{200}{x^2 + 4x + 25}$$



Area $= \frac{1}{2}$(base)(height)

$$A = \frac{1}{2} \cdot x \cdot f(x)$$

$$A = \frac{1}{2} \cdot x \cdot \frac{200}{x^2+4x+25}$$

$$A = \frac{100x}{x^2+4x+25}$$

maximize A on $(0, \infty)$

$$A' = \frac{100(x^2+4x+25) - 100x(2x+4)}{(x^2+4x+25)^2}$$

$$A' = \frac{100(25-x^2)}{(x^2+4x+25)^2} = \frac{100(5+x)(5-x)}{(x^2+4x+25)^2}$$

$x > 0$ and $A' = 0 \Rightarrow x = 5$

values of A'

$$\underset{0}{(} \quad + \quad \underset{5}{0} \quad - \quad \rightarrow x$$

abs. max. area when $x = 5$ is $A(5) = \frac{500}{70} = \frac{50}{7}$