Math 220 (section AD?) **Quiz 6** **Fall 2019**

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (3 points) Suppose that p is a function of w which satisfies the following two conditions.

   - $\frac{dp}{dw} = 0.5p$
   - $p\,(ln(25)) = 20$

   Evaluate $p\,(ln(81))$ and simplify your answer.

$$\boxed{\text{recall } \frac{dy}{dx} = ky \Rightarrow y = Ce^{kx}}$$

thus, $\frac{dp}{dw} = 0.5p \Rightarrow p = Ce^{0.5w}$

$$p(\ln(25)) = 20 \Rightarrow 20 = Ce^{0.5\ln(25)}$$

$$20 = Ce^{0.5\ln(5^2)}$$

$$20 = Ce^{(0.5)\cdot 2\ln(5)}$$

$$20 = Ce^{\ln(5)}$$

$$20 = C\cdot 5$$

$$4 = C$$

thus, $p = 4e^{0.5w}$

$$p(\ln(81)) = 4e^{0.5\ln(81)}$$

$$= 4e^{0.5\ln(9^2)}$$

$$= 4e^{(0.5)\cdot 2\ln(9)}$$

$$= 4e^{\ln(9)} = 4\cdot 9 = 36$$

2. (3 points) For $t \geq 0$, the position in meters of a particle is given by

$$s(t) = \frac{t^3}{3} - 2t^2 + 10t + 17$$

where t is measured in seconds.

What is the particle's acceleration at the moment when the particle's velocity is 15 m/s ? Use correct units in your final answer.

position —— $s(t) = \frac{t^3}{3} - 2t^2 + 10t + 17$

velocity —— $s'(t) = t^2 - 4t + 10$

acceleration — $s''(t) = 2t - 4$

velocity is 15 m/s

$t^2 - 4t + 10 = 15$

$t^2 - 4t - 5 = 0$

$(t-5)(t+1) = 0$

$t = 5$ or $t = -1$, but only considering $t \geq 0$

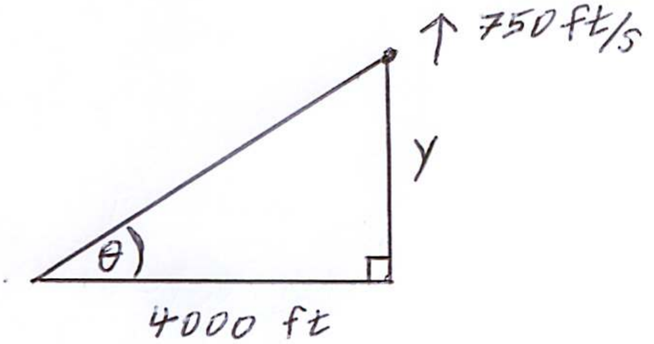
$t = 5$

acceleration is

$s''(5) = 2 \cdot 5 - 4$

$= 6 \ m/s^2$

3. (4 points) A camera is positioned on the ground 4000 feet from the base of a rocket launching pad. The angle of elevation of the camera has to change at the correct rate in order to keep the rocket in sight. Suppose the rocket rises vertically and its speed is 750 feet per second when it has risen 3000 feet. How quickly is the camera's angle of elevation increasing at that moment?



Given: $\left.\frac{dy}{dt}\right|_{y=3000\text{ ft}} = 750$ ft/s

want: $\left.\frac{d\theta}{dt}\right|_{y=3000\text{ ft}}$

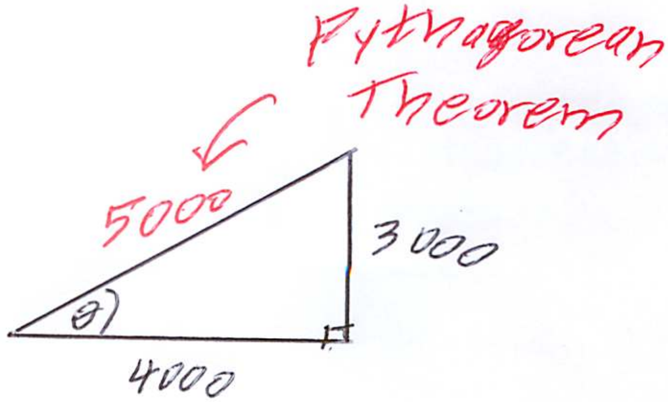
$$\tan(\theta) = \frac{y}{4000} = \frac{1}{4000}y$$

$$\frac{d}{dt}(\tan(\theta)) = \frac{d}{dt}\left(\frac{1}{4000}y\right)$$

$$\sec^2(\theta)\frac{d\theta}{dt} = \frac{1}{4000}\frac{dy}{dt}$$

at $y = 3000$ ft, we have

$$\sec(\theta) = \frac{5000}{4000} = \frac{5}{4}$$



$$\left(\frac{5}{4}\right)^2\frac{d\theta}{dt} = \frac{1}{4000}\cdot 750$$

$$\frac{d\theta}{dt} = \frac{750}{4000}\cdot\left(\frac{4}{5}\right)^2$$

$$= \frac{75}{400}\cdot\frac{16}{25}$$

$$= \frac{3}{25}\ \text{rad/s}$$

<u>note</u>: your memorized derivative formulas are only valid when angles are measured in radians