Math 220 (section AD?)

Name: Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) Write an equation for the line which is tangent to \( f(x) = x^4 + x^3 - 2x^2 \) at its negative \( x \)-intercept.

   Set \( y = 0 \) to find \( x \)-intercept:
   
   \[
   0 = x^4 + x^3 - 2x^2 \\
   0 = x^2(x^2 + x - 2) \\
   0 = x^2(x + 2)(x - 1) \\
   x = 0, x = -2, x = 1 \\
   \text{neg. } x-\text{int is } -2
   \]

   Point: \((-2, 0)\)
   Slope: \(-22\)

   Tangent line: \( y - 0 = -22(x - (-2)) \)
   \[
   y = -22x - 44
   \]

2. (2 points) What is the slope of the following curve at its \( y \)-intercept? Simplify your answer.

   \[
   g(x) = 5\sin(x) - 2e^x + 16\cos(x) + 10x^2 + 4x - 9
   \]

   \[
   g'(x) = 5\cos(x) - 2e^x - 16\sin(x) + 20x + 4
   \]

   \[
   g'(0) = 5 \cdot \cos(0) - 2e^0 - 16\sin(0) + 20(0) + 4
   \]

   \[
   g'(0) = 5 - 2 - 0 + 0 + 4
   \]

   Slope = \( 7 \)
3. (2 points each) Using Leibniz notation (i.e., $\frac{dy}{dx}$, $\frac{dp}{dt}$, etc.), find derivatives for each of the following functions. Your answer to part (a) must be simplified.

(a) \[ w = \left( \frac{\sqrt{p}}{p^{3/2}} \right)^{-12} = \left( \frac{p^{1/2}}{p^{3/2}} \right)^{-12} = \left( p^{\frac{1}{2} - \frac{3}{2}} \right)^{-12} = \left( p^{\frac{-2}{6}} \right)^{-12} = p^{\frac{2}{6} \cdot 12} = p^{14} \]

\[ \frac{dw}{dp} = 14p^{13} \]

(b) \[ R = v^5 \csc(v) + \ln (5\pi^3 + 2e^4) = v^5 \csc(v) + \text{constant} \]

\[ \frac{dR}{dv} = \frac{d}{dv}(v^5 \csc(v)) + v^5 \frac{d}{dv}(\csc(v)) + 0 \]

\[ \frac{dR}{dv} = 5v^4 \csc(v) + v^5 \left( -\csc(v) \cot(v) \right) \]

(c) \[ \theta = \frac{5e^t + \sqrt{t}}{9t + \tan(t)} \]

\[ \frac{d\theta}{dt} = \frac{\frac{d}{dt}(5e^t + \sqrt{t})(9t + \tan(t)) - (5e^t + \sqrt{t})\frac{d}{dt}(9t + \tan(t))}{(9t + \tan(t))^2} \]

\[ \frac{d\theta}{dt} = \frac{(5e^t + \sqrt{t})(9t + \tan(t)) - (5e^t + \sqrt{t})(9 + 5e^t \sec^2(t))}{(9t + \tan(t))^2} \]