Math 220 (section AD?) **Quiz 4** **Fall 2019**

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) Write an equation for the line which is tangent to $f(x) = x^4 + x^3 - 2x^2$ at its negative x-intercept.

Set $y=0$ to find x-intercept

$0 = x^4 + x^3 - 2x^2$

$0 = x^2(x^2 + x - 2)$

$0 = x^2(x+2)(x-1)$

$x=0,\ x=-2,\ x=1$

neg. x-int is -2

$f'(x) = 4x^3 + 3x^2 - 4x$

$f'(-2) = 4(-2)^3 + 3(-2)^2 - 4(-2)$

$= -32 + 12 + 8$

$= -12$

point: $(-2, 0)$

slope: $f'(-2) = -12$

tangent line: $y - 0 = -12(x - (-2))$

$y = -12x - 24$

2. (2 points) What is the slope of the following curve at its y-intercept? Simplify your answer.

$$g(x) = 5\sin(x) - 2e^x + 16\cos(x) + 10x^2 + 4x - 9$$

$\boxed{\text{at y-intercept, } x=0}$

$g'(x) = 5\cos(x) - 2e^x - 16\sin(x) + 20x + 4$

$g'(0) = 5\cos(0) - 2e^0 - 16\sin(0) + 20(0) + 4$

$= 5 - 2 - 0 + 0 + 4$

$= 7$

slope $= 7$

3. (2 points each) Using Leibniz notation (i.e., $\frac{dy}{dx}$, $\frac{dP}{dt}$, etc.), find derivatives for each of the following functions. Your answer to part (a) must be simplified.

(a) $w = \left(\frac{\sqrt[3]{p}}{p\sqrt{p}}\right)^{-12}$ $= \left(\frac{p^{1/3}}{p^{3/2}}\right)^{-12} = \left(p^{\frac{1}{3}-\frac{3}{2}}\right)^{-12} = \left(p^{\frac{2}{6}-\frac{9}{6}}\right)^{-12}$

$= \left(p^{-\frac{7}{6}}\right)^{-12} = p^{-\frac{7}{6}(-12)} = p^{14}$

$\frac{dw}{dp} = 14p^{13}$

(b) $R = v^5 \csc(v) + \ln(5\pi^3 + 2e^4)$ $= v^5 \csc(v) + \text{constant}$

$\frac{dR}{dv} = \frac{d}{dv}(v^5)\cdot\csc(v) + v^5 \cdot \frac{d}{dv}(\csc(v)) + 0$

$\frac{dR}{dv} = 5v^4 \cdot \csc(v) + v^5 \cdot (-\csc(v)\cot(v))$

(c) $\theta = \frac{5e^t + \sqrt{t}}{9t + \tan(t)}$

$$\frac{d\theta}{dt} = \frac{\frac{d}{dt}(5e^t + \sqrt{t})\cdot(9t + \tan(t)) - (5e^t + \sqrt{t})\frac{d}{dt}(9t + \tan(t))}{(9t + \tan(t))^2}$$

$$\frac{d\theta}{dt} = \frac{(5e^t + \frac{1}{2}t^{-1/2})(9t + \tan(t)) - (5e^t + \sqrt{t})(9 + \sec^2(t))}{(9t + \tan(t))^2}$$