Math 220 (section AD?) Quiz 3 Fall 2019

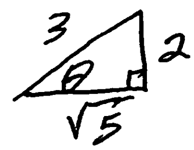
Name Solutions

- 20 minutes
- No calculators
- Show sufficient work

We will learn l'Hospital's Rule and other shortcuts for obtaining limits later. For now you are not allowed to use these approaches.

1. (2 points) Evaluate $\tan(\arcsin(2/3))$.

Let $\theta = \arcsin(2/3)$
Then $\sin(\theta) = 2/3$
and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$,
Do you see why θ is acute?



$\tan(\arcsin(2/3)) = \tan(\theta)$
$= \frac{2}{\sqrt{5}}$

2. (2 points each) Evaluate the following limits. For infinite limits, you must clearly show whether the limit is ∞ or $-\infty$.

(a) $\lim\limits_{x\to 2} \frac{x^2-3x+2}{x^2-4}$ $\begin{matrix}\to 0 \\ \to 0\end{matrix}$ $= \lim\limits_{x\to 2} \frac{(x-2)(x-1)}{(x-2)(x+2)}$

$= \lim\limits_{x\to 2} \frac{x-1}{x+2}$

$= \frac{2-1}{2+2}$

$= \frac{1}{4}$

(b) $\lim\limits_{x\to 2}\dfrac{\sqrt{x^2+5}-3}{x-2}$ $\begin{matrix}\nearrow 0 \\ \searrow 0\end{matrix}$

$$= \lim_{x\to 2} \frac{\sqrt{x^2+5}-3}{x-2}\cdot\frac{\sqrt{x^2+5}+3}{\sqrt{x^2+5}+3}$$

$$= \lim_{x\to 2} \frac{(\sqrt{x^2+5})^2-(3)^2}{(x-2)(\sqrt{x^2+5}+3)}$$

$$= \lim_{x\to 2} \frac{x^2+5-9}{(x-2)(\sqrt{x^2+5}+3)}$$

$$= \lim_{x\to 2} \frac{x^2-4}{(x-2)(\sqrt{x^2+5}+3)}$$

$$= \lim_{x\to 2} \frac{(x-2)(x+2)}{(x-2)(\sqrt{x^2+5}+3)}$$

$$= \lim_{x\to 2} \frac{x+2}{\sqrt{x^2+5}+3}$$

$$= \frac{2+2}{\sqrt{2^2+5}+3} = \frac{4}{6} = \frac{2}{3}$$

(c) $\lim\limits_{x\to e^+}\dfrac{1-\ln(\sqrt{x})}{1-\ln(x)}$

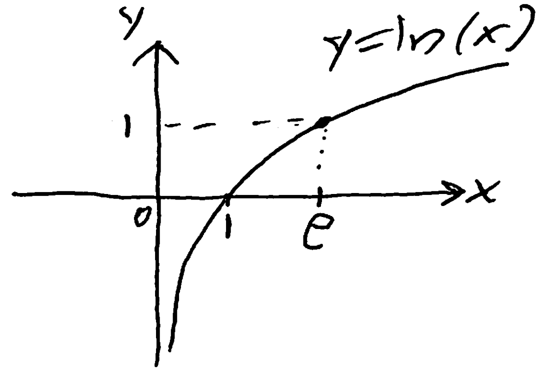
$$1-\ln(\sqrt{e}) = 1-\ln(e^{1/2}) = 1-\tfrac{1}{2} = \tfrac{1}{2}$$

$$1-\ln(e) = 1-1 = 0$$

$$\lim_{x\to e^+} \frac{1-\ln(\sqrt{x}) \to 1/2}{1-\ln(x) \to 0^-} = -\infty$$

note: From graph of $\ln(x)$, if $x>e$, then $\ln(x)>1$

Thus, if $x>e$, $1-\ln(x)<0$

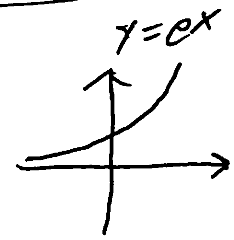


3. (2 points) Write an equation for each horizontal asymptote on the graph of the given function. Use limits to justify your answer.

$$f(x) = \frac{12e^{-x}+35}{5+3e^{-x}} = \frac{\frac{12}{e^x}+35}{5+\frac{3}{e^x}} \cdot \frac{e^x}{e^x} = \frac{12+35e^x}{5e^x+3}$$

$$\lim_{x\to-\infty} \frac{12+35e^x}{5e^x+3} = \frac{12}{3} = 4$$

since $e^x \to 0$ as $x \to -\infty$



$$\lim_{x\to\infty} \frac{12+35e^x}{5e^x+3} \quad \left(\frac{\to\infty}{\to\infty}\right) = \lim_{x\to\infty} \frac{12+35e^x}{5e^x+3} \cdot \frac{1/e^x}{1/e^x}$$

$$= \lim_{x\to\infty} \frac{\frac{12}{e^x}+35}{5+\frac{3}{e^x}}$$

$$= \frac{35}{5}$$

$$= 7$$

since $\frac{K}{e^x} \to 0$ as $x \to \infty$ for any number K

The horizontal asymptotes are $y=4$ and $y=7$