Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (4 points) A bacterial culture starts with 300 bacteria and doubles in size every 4 hours.

(a) Find a formula for the number of bacteria after t hours.

Let A be the number of bacteria at t hours,

t	A
0	300
4	600
8	1200
12	2400
⋮	⋮

$$A = C \cdot a^t$$

$$300 = C \cdot a^0 \Rightarrow C = 300$$

$$A = 300 \cdot a^t$$

$$600 = 300 \cdot a^4 \Rightarrow a^4 = 2 \Rightarrow a = 2^{1/4}$$

$$A = 300 \cdot (2^{1/4})^t$$

$$A = 300 \cdot 2^{t/4}$$

(b) At what time is the population equal to 1500 ?

$$1500 = 300 \cdot 2^{t/4}$$

$$5 = 2^{t/4}$$

$$\ln(5) = \ln(2^{t/4})$$

$$\ln(5) = \frac{t}{4} \cdot \ln(2)$$

$$t = \frac{4 \ln(5)}{\ln(2)} \text{ hours}$$

2. (3 points) Determine all values of x which satisfy the equation below.

$$2\ln(9) + 3e^{x} = \ln(81) + 24e^{-2x}$$

$$\ln(9^2) + 3e^{x} = \ln(81) + 24e^{-2x}$$

$$\ln(81) + 3e^{x} = \ln(81) + 24e^{-2x}$$

$$3e^{x} = 24e^{-2x}$$

$$\frac{e^{x}}{e^{-2x}} = \frac{24}{3}$$

$$e^{3x} = 8$$

$$\ln(e^{3x}) = \ln(8)$$

$$3x = \ln(8)$$

$$x = \frac{1}{3}\ln(8)$$

$$= \frac{1}{3}\ln(2^3)$$

$$= \frac{1}{3}\cdot 3\ln(2)$$

$$= \ln(2)$$

3. (3 points) The function $h(x) = \sqrt{3 + \ln(5e^{2x} + 9)}$ is one-to-one. Determine a formula for its inverse $h^{-1}(x)$.

$$y = \sqrt{3 + \ln(5e^{2x} + 9)}$$

$$x = \sqrt{3 + \ln(5e^{2y} + 9)} \quad \text{(switch x \& y)}$$

$$x^2 = 3 + \ln(5e^{2y} + 9)$$

$$x^2 - 3 = \ln(5e^{2y} + 9)$$

$$e^{x^2-3} = e^{\ln(5e^{2y}+9)}$$

$$e^{x^2-3} = 5e^{2y} + 9$$

$$e^{x^2-3} - 9 = 5e^{2y}$$

$$\frac{e^{x^2-3} - 9}{5} = e^{2y}$$

$$\ln\left(\frac{e^{x^2-3} - 9}{5}\right) = \ln(e^{2y})$$

$$\ln\left(\frac{e^{x^2-3} - 9}{5}\right) = 2y$$

$$\frac{1}{2}\ln\left(\frac{e^{x^2-3} - 9}{5}\right) = y$$

$$\boxed{h^{-1}(x) = \frac{1}{2}\ln\left(\frac{e^{x^2-3} - 9}{5}\right)}$$