Math 220 (section AD) **Quiz 10** **Fall 2019**

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) Fill in the missing information for the following two theorems.

$\boxed{\textbf{Mean Value Theorem}}$ Let f be a function that satisfies the following two hypotheses.

(1) f is continuous on the closed interval $[a, b]$.

(2) f is differentiable on the open interval (a, b).

Then there is a number c in (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

$\boxed{\textbf{Rolle's Theorem}}$ Let f be a function that satisfies the following three hypotheses.

(1) f is continuous on the closed interval $[a, b]$.

(2) f is differentiable on the open interval (a, b).

(3) $f(a) = f(b)$.

Then there is a number c in (a, b) such that $f'(c) = 0$.

2. (2 points) Evaluate the definite integral. Simplify your answer.

$$\int_1^{e^2} \frac{24}{x\sqrt{9+8\ln(x)}}\,dx$$

$$= \int_1^{e^2} \frac{3}{\sqrt{9+8\ln(x)}} \cdot \frac{8}{x} dx$$

$$= \int_9^{25} \frac{3}{\sqrt{u}} du$$

$$= \int_9^{25} 3u^{-1/2} du$$

$$= 6u^{1/2}\Big|_9^{25}$$

$$= 6\sqrt{25} - 6\sqrt{9}$$

$$= 6\cdot 5 - 6\cdot 3 = 12$$

substitution

$u = 9 + 8\ln(x)$

$du = \frac{8}{x} dx$

$x = 1 \Rightarrow u = 9 + 8\ln(1)$
$= 9$

$x = e^2 \Rightarrow u = 9 + 8\ln(e^2)$
$= 25$

3. (2 points) Evaluate the indefinite integral.

$$\int \frac{10e^{2x}\cos(e^{2x})}{\sin^2(e^{2x})+1}\,dx$$

$$= \int \frac{5\cos(e^{2x})}{\sin^2(e^{2x})+1} \cdot 2e^{2x} dx$$

$$= \int \frac{5\cos(u)}{\sin^2(u)+1} du$$

$$= 5\int \frac{1}{\sin^2(u)+1} \cdot \cos(u) du$$

$$= 5\int \frac{1}{w^2+1} dw$$

$$= 5\arctan(w) + C$$

$$= 5\arctan(\sin(u)) + C$$

$$= 5\arctan(\sin(e^{2x})) + C$$

substitution #1

$u = e^{2x}$

$du = 2e^{2x} dx$

substitution #2

$w = \sin(u)$

$dw = \cos(u) du$

4. (2 points) Evaluate the indefinite integral.

$$\int \frac{4x^{11}}{(x^4+2)^3}\,dx$$

Substitution

$u = x^4 + 2$

$du = 4x^3\,dx$

note:

$u = x^4 + 2 \Rightarrow x^4 = u - 2$

$$= \int \frac{x^8}{(x^4+2)^3} \cdot 4x^3\,dx$$

$$= \int \frac{(x^4)^2}{(x^4+2)^3} \cdot 4x^3\,dx$$

$$= \int \frac{(u-2)^2}{u^3}\,du$$

$$= \int \frac{u^2 - 4u + 4}{u^3}\,du$$

$$= \int \left(\frac{u^2}{u^3} - \frac{4u}{u^3} + \frac{4}{u^3}\right)du$$

$$= \int \left(\frac{1}{u} - 4u^{-2} + 4u^{-3}\right)du$$

$$= \ln|u| + 4u^{-1} - 2u^{-2} + C$$

$$= \ln(x^4+2) + \frac{4}{x^4+2} - \frac{2}{(x^4+2)^2} + C$$

5. (2 points) Let **R** be the finite region bounded by the given functions. In the following way, set up <u>but do not evaluate</u> definite integrals which represent the area of the region **R**.

$y = 10\sqrt{x}$

$y = 5x$

intersection

$5x = 10\sqrt{x}$

$25x^2 = 100x$

$25x^2 - 100x = 0$

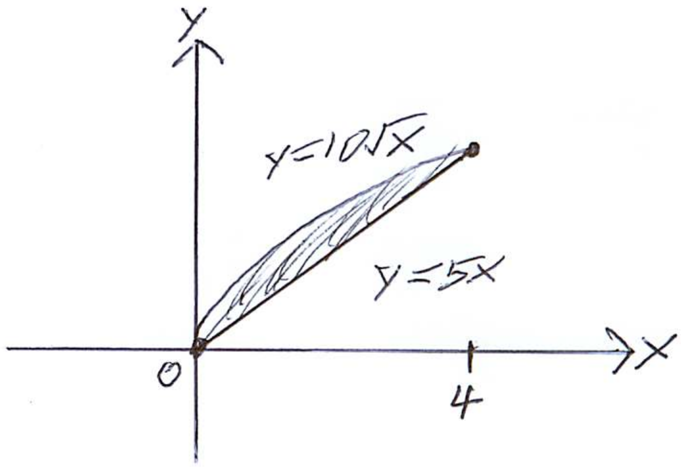
$25x(x-4) = 0$

$x = 0$, $y = 0$; $x = 4$, $y = 20$

(a) Integrate with respect to x.

$$\text{area} = \int_{x_{min}}^{x_{max}} (y_{top} - y_{bottom})\,dx$$

$$= \int_0^4 (10\sqrt{x} - 5x)\,dx$$



(b) Integrate with respect to y. (The integrands in parts (a) and (b) should be different.)

To integrate with respect to y, we must solve for x.

$y = 5x \Rightarrow x = \frac{y}{5}$

$y = 10\sqrt{x} \Rightarrow x = \frac{y^2}{100}$

$$\text{area} = \int_{y_{min}}^{y_{max}} (x_{right} - x_{left})\,dy$$

$$= \int_0^{20} \left(\frac{y}{5} - \frac{y^2}{100}\right) dy$$