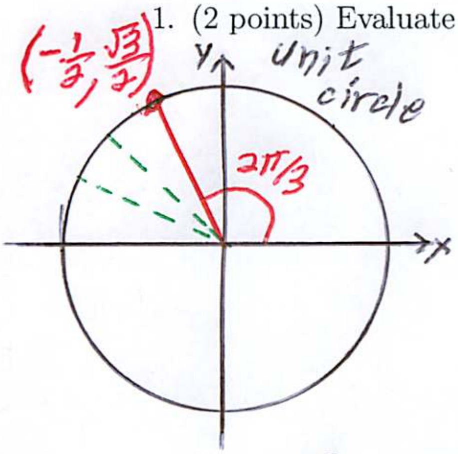
Math 220 (section AD?) Quiz 1 Fall 2019

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) Evaluate and simplify the following quantity.



$$8\sin^2(2\pi/3) + 9\csc^2(\pi/13) - 9\cot^2(\pi/13)$$

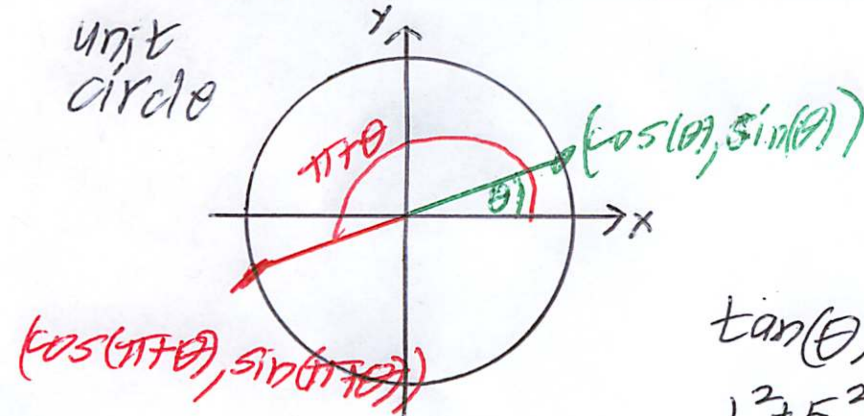
From the unit circle, $\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

we also use the identity

$\cot^2(\theta) + 1 = \csc^2(\theta)$

$$8\sin^2\left(\tfrac{2\pi}{3}\right) + 9\csc^2\left(\tfrac{\pi}{13}\right) - 9\cot^2\left(\tfrac{\pi}{13}\right)$$
$$= 8\left(\tfrac{\sqrt{3}}{2}\right)^2 + 9\left(\cot^2\left(\tfrac{\pi}{13}\right) + 1\right) - 9\cot^2\left(\tfrac{\pi}{13}\right)$$
$$= 8\left(\tfrac{3}{4}\right) + 9$$
$$= 15$$

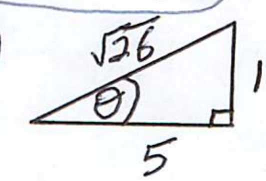
2. (2 points) Given an acute angle θ for which $\tan(\theta) = 1/5$, evaluate $\sin(\pi + \theta)$.



From the unit circle we see that $\sin(\pi+\theta) = -\sin(\theta)$

$\tan(\theta) = \frac{1}{5}$ $\left(\frac{opp}{adj}\right)$

$1^2 + 5^2 = (hyp)^2 \Rightarrow hyp = \sqrt{26}$



$\sin(\theta) = \frac{1}{\sqrt{26}}$ $\left(\frac{opp}{hyp}\right)$

$$\sin(\pi+\theta) = -\sin(\theta) = -\frac{1}{\sqrt{26}}$$

3. (3 points) State the domain of the function using interval notation.

$$f(x) = \frac{9 + \sqrt[4]{4000 - 10x^2}}{2 + \sqrt[3]{4x + 12}}$$

From $\sqrt[4]{4000-10x^2}$, we see that $4000-10x^2 \geq 0$

$-10x^2 \geq -4000$

$x^2 \leq 400$

$\sqrt{x^2} \leq \sqrt{400}$

$|x| \leq 20$

$-20 \leq x \leq 20$

The denominator $=0$ when

$2+\sqrt[3]{4x+12} = 0$

$\sqrt[3]{4x+12} = -2$

$4x+12 = (-2)^3 = -8$

$4x = -20$

$x = -5$

Thus $x \neq -5$

Domain of $f(x)$

$[-20,-5) \cup (-5,20]$

4. (3 points) Use the definitions of even and odd functions to prove whether the following function is even, odd or neither.

$$f(x) = x^9 \sin\left(2x^3 + 4x^5\right)$$

$f(-x) = (-x)^9 \sin(2(-x)^3 + 4(-x)^5)$

$= -x^9 \sin(2(-x^3) + 4(-x^5))$

$= -x^9 \sin(-2x^3 - 4x^5)$

$= -x^9 \sin(-(2x^3+4x^5))$

$= -x^9 \cdot -\sin(2x^3+4x^5)$ since sin is an odd Function

$= x^9 \sin(2x^3+4x^5)$

$= f(x)$

f is even