Solution to the dog and frisbee problem

A frisbee has been thrown into the water from a point along a straight shoreline. A dog at that point can run at 25 feet per second on land along the shoreline for 20 feet, and then make a right angle turn into the water to swim at 7 feet per second for 12 feet to retrieve the frisbee. However, the dog can retrieve the frisbee more quickly by entering the water sooner. At what point should the dog enter the water to minimize the total time needed to retrieve the frisbee?

\[ \text{distance} = \text{rate} \times \text{time} \]

\[ \text{time} = \frac{\text{distance}}{\text{rate}} \]

rate on land = 25 ft/s
rate in water = 7 ft/s

Total time = (time on land) + (time in water)

\[ T = \frac{x}{25} + \frac{\sqrt{(20-x)^2 + 12^2}}{7} \]

we will minimize \( T \) for \( x \) in \([0, 20]\)

\[ T' = 0.04 + \frac{1}{2} \left( \frac{(20-x)^2 + 12^2}{7} \right)^{-\frac{1}{2}} - \frac{2}{7}(20-x)(-1) \]

\[ T' = \frac{1}{25} + \frac{(20-x)(-1)}{7\sqrt{(20-x)^2 + 12^2}} \]

set \( T' = 0 \)
\[
0 = \frac{1}{25} - \frac{20-x}{7\sqrt{(20-x)^2+12^2}}
\]

\[
\frac{20-x}{7\sqrt{(20-x)^2+12^2}} = \frac{1}{25}
\]

\[25(20-x) = 7\sqrt{(20-x)^2+12^2}\]

\[(25(20-x))^2 = (7\sqrt{(20-x)^2+12^2})^2\]

\[625(20-x)^2 = 49((20-x)^2+12^2)\]

\[(625-49)(20-x)^2 = 49 \cdot 12^2\]

\[576(20-x)^2 = 49 \cdot 12^2\]

\[\sqrt{576(20-x)^2} = \sqrt{49 \cdot 12^2}\]

\[24(20-x) = 7 \cdot 12\]

\[2(20-x) = 7\]

\[x = 20 - \frac{21}{2} = \frac{33}{2} = 16.5\]

<table>
<thead>
<tr>
<th>x</th>
<th>T = \frac{x}{25} + \frac{\sqrt{(20-x)^2+12^2}}{7}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.33197</td>
</tr>
<tr>
<td>16.5</td>
<td>2.44571 (minimum time)</td>
</tr>
<tr>
<td>20</td>
<td>2.51429</td>
</tr>
</tbody>
</table>

The dog should run on land for 16.5 ft and then swim to the frisbee.